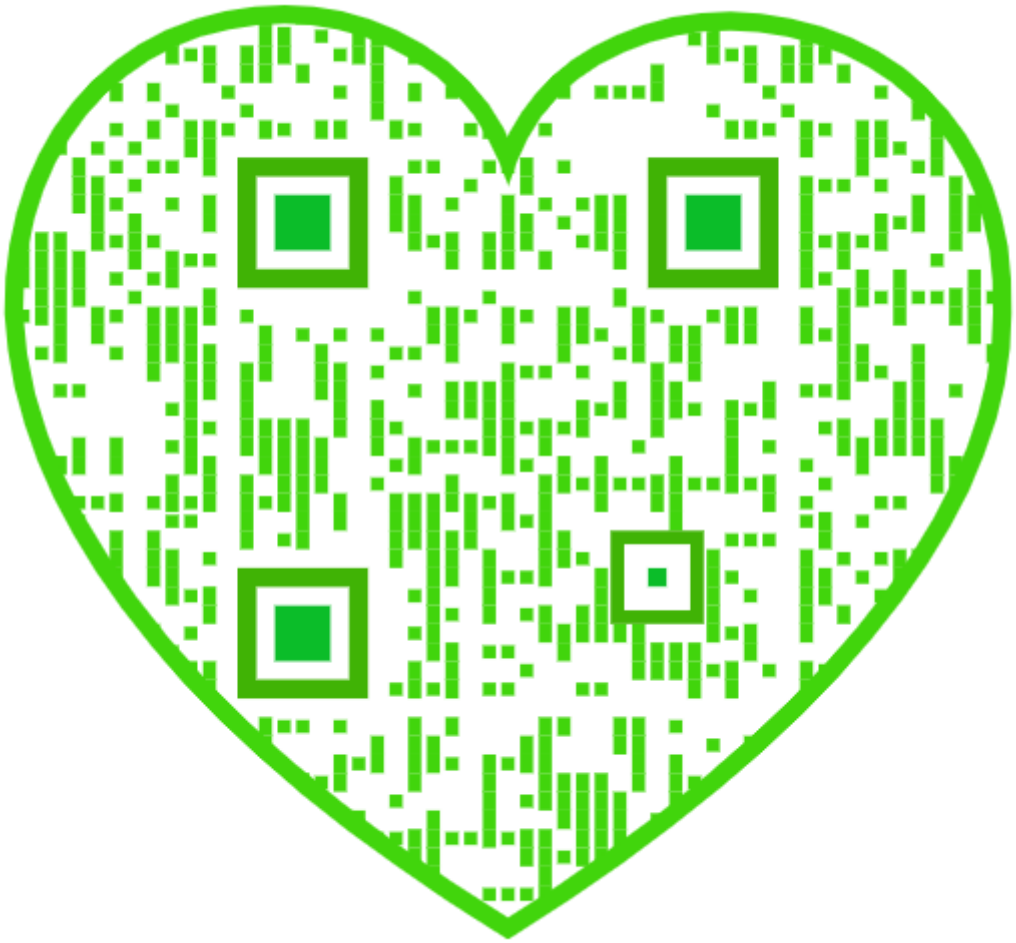

AN

ELEMENTARY

ARITHMETIC

BY

D. B. HAGAR, PH.D.,

PRINCIPAL OF STATE NORMAL SCHOOL, SALEM, MASS.

PHILADELPHIA

COWPERTHWAIT & CO.

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INTRODUCTION.

THE purpose of this manual is to facilitate the advance of young learners in the science of numbers by gradual steps.

The lessons are intended to secure that normal development and discipline of the reasoning powers, and those correct habits of investigation, which alone form a sure foundation for progress in any branch of knowledge.

The principles and rules have been carefully established by induction. The plan has been to make the reasons for each process entirely clear, and to enable the learner to state them in concise language.

Mental and written exercises admitting substantially of the same solution have been combined, so as to render unnecessary the use of a separate mental arithmetic, and otherwise to abridge advantageously the ordinary course of arithmetical study.

Pictorial illustrations, from original designs, have been freely introduced, with the view of making some parts of the subjects treated more easily understood, through the medium of the eye.

It is hoped that this work, which is complete in itself, may satisfactorily meet the wants of intermediate classes in graded schools; and also may prove useful in many district schools, in which the attendance is too limited to warrant the use of a more extended treatise.

SUGGESTIONS TO TEACHERS.

1. MATERIAL objects should be used as illustrations far enough to make sure that the pupils clearly understand the value of numbers, how numbers are composed in addition, how they are separated in subtraction, how multiplication and division are performed, and how the elementary tables are constructed. When these things are comprehended, material objects should be dispensed with, and a thorough knowledge of the tables should be relied upon for the requisite results.

2. The tables should be made so familiar that when any two numbers are named, the result of a desired operation upon them shall, by the power of repeated association, instantly flash upon the mind.

3. Care should be taken that the definitions are clearly understood before they are learned.

4. The attention of pupils should be directed to the successive steps taken in the solution of the problems first given under any subject, and each pupil should be required to state the first step, the second step, and so on until all the steps are named and recorded on slates or blackboard. These steps should be combined, and thus the mode of building up a rule be made clear. The rule should be regarded, not as a guide to the solution of problems, but as a concise statement of what the pupils have already learned to do.

5. In addition, pupils should usually avoid naming the numbers to be added, but should give only the successive results. They should have much practice in adding and subtracting by 2's, 3's, 4's, etc. In multiplication, they may profitably use either form of expression—2 times 2, 3 times 2, 4 times 2, etc.; or, two 2's, three 2's, four 2's, etc. Sometimes one of these expressions is preferable, sometimes the other.

6. *The explanations given are not to be committed to memory. The definitions and principles, having been fully comprehended, ought to be fixed in the memory. The rules may or may not be learned, as teachers shall prefer.*

7. Fractions should be amply illustrated by material objects, attention being specially called to the number and the size of the parts into which a thing is divided.

8. Care should be taken that the explanations given by pupils are logical in order and accurate in expression.

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ELEMENTARY ARITHMETIC.

SECTION I.

NUMBERS.

ARTICLE 1.—1. Arthur has one white rabbit and one gray rabbit. How many rabbits has he?

2. Arthur has two rabbits, and his sister has one. How many rabbits have both of them?

3. Jane had two books, and her father gave her two more. How many books had she then?

4. If you have three cents in one pocket, and two cents in another, how many have you in both?

5. James gave four cents for an orange, and two cents for an apple. How many cents did he give for both?

6. If you count the fingers and thumb on your right hand, how many will you find them to be?

7. How many are six books and two books?

8. Arthur has five lead pencils and four slate pencils. How many pencils has he?

9. Six trees and three trees are how many trees?

10. How many are one and one? Two and one? Three and one? Four and one?

11. How many are five and one? Six and one? Seven and one? Eight and one? Nine and one?

12. How many are two and two? Three and two? Four and two? Five and two? Six and two?

13. How many are seven and two? Eight and two? Three and two? Three and three? Four and three?

14. How many are five and three? Six and three? Seven and three? Four and four?

15. How many are five and four? Six and four? Five and five?

16. Count from one to ten. Count the fingers and thumbs which you have on both hands.

17. If you had five books, and should have three more given you, how many books would you then have?

18. How many are five and three? Three and five?

19. How many are six and two? Eight and two? Four and three? Seven and three?

20. If you have six books, and have four more given you, how many books will you then have?

2.—1. How many ones in a collection of one one and one one? Of two ones and one one?

2. How many ones in a collection of nine ones and one one? Of ten ones and one one?

3. How many are nine and two? Eight and three? Nine and three? Seven and four?

4. How many dollars are six dollars and four dollars? Seven dollars and three dollars?

5. How many are eight and four? Six and five? Six and six?

6. How many are eight and six? Nine and six? Seven and seven? Eight and seven?

7. How many are eight and eight? Nine and eight? Nine and nine?

8. How many are ten and five? Ten and four? Seven and four? Nine and three?

9. How many are five and five? Nine and five? Seven and six? Eight and six?

10. How many are seven and seven? Ten and six? Eight and eight?

11. How many are six and ten? Nine and six? Seven and eight? Eight and nine?

12. How many are eight and ten? Ten and nine? Nine and ten? Ten and ten? Count from one to twenty.

13. How many tens are one ten and one ten? Two tens and three tens? Count from one to thirty.

14. How many tens are four tens and three tens? Six tens and four tens? Count from one to one hundred.

DEFINITIONS.

3. A **Unit** is one, or a single thing of any kind.

4. A **Number** is a unit, or a collection of units.

5. A **Figure** is a character used to express a number.

Each of the first nine numbers is expressed by a single figure, thus—

PRINTED.	1,	2,	3,	4,	5,	6,	7,	8,	9.
WRITTEN.	1	2	3	4	5	6	7	8	9
NAMED.	One,	two,	three,	four,	five,	six,	seven,	eight,	nine.

The figure 0, which is called **Zero,** or **Cipher,** expresses the absence of number.

1 ten is named **Ten.**
2 tens are named **Twenty.**
3 tens are named **Thirty.**
4 tens are named **Forty.**
5 tens are named **Fifty.**
6 tens are named **Sixty.**
7 tens are named **Seventy.**
8 tens are named **Eighty.**
9 tens are named **Ninety.**
10 tens are named **One hundred.**

When a number is expressed by two figures, side by side, the figure on the right, or in the first place, expresses **Ones,** and the figure on the left, or in the second place, expresses **Tens.**

Thus, 57 expresses 5 tens 7 ones, or fifty-seven.

WRITTEN EXERCISES.

6. Copy and name the number expressed—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
19	17	16	62	75	93
21	70	74	53	42	37
23	62	57	77	57	82
36	91	80	66	90	28
41	82	11	46	28	69
55	73	85	64	82	99

SECTION II.

NUMERATION AND NOTATION.

7.—1. By combining what two figures do we express ten, or 1 ten 0 ones? Eleven, or 1 ten 1 one?

2. By combining what two figures do we express twelve, or 1 ten 2 ones? Thirteen, or 1 ten 3 ones?

3. By combining what two figures do we express nineteen, or 1 ten 9 ones? Ninety-one, or 9 tens 1 one?

4. By combining what two figures do we express seventy-five, or 7 tens 5 ones? Fifty-seven, or 5 tens 7 ones?

5. By combining what two figures do we express eighty, or 8 tens 0 ones? Ninety-three, or 9 tens 3 ones?

DEFINITIONS.

8. **Numeration** is the method of naming numbers.

9. **Notation** is the method of writing numbers.

10. In **Naming Numbers,** ten ones are named **one ten,** ten tens are named **one hundred,** and so on.

11. A figure written alone, or at the left of a point (.), called the **Decimal Point,** expresses ones, or **Primary Units.**

12. **Orders of Units** are expressed by the successive figures written side by side to express number.

When a number is expressed by three figures, the first figure at the left of the decimal point expresses units of the **First Order,** or **Ones;** the second figure at the left of the point expresses units of the **Second Order,** or

Tens; the third figure at the left of the point expresses units of the **Third Order**, or **Hundreds**, and so on.

Thus, 365 expresses 5 units of the first order, 6 units of the second order, 3 units of the third order, or three hundred sixty-five.

13. In **Reading Numbers**, each of the three orders of figures at the left of the decimal point expresses a **Class** or **Period** of units, with a distinct name, having ones, tens and hundreds, as shown in the following

NUMERATION TABLE.

PERIODS.	Millions.			Thousands.			Units.			
ORDERS.	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.	Decimal Point.
NUMBER.	3	5	6,	7	8	9,	4	0	3	.,

where the number expressed by the figures is three hundred fifty-six *millions* seven hundred eighty-nine *thousands* four hundred three.

14. The **Comma** (,) is used to separate the classes or periods, and the **Decimal Point** (.) to mark the order of primary units.

When the decimal point is not expressed it is always understood.

Thus, 32. or 32 expresses thirty-two.

15. A **Solution** is the process of answering a question requiring computation.

16. A **Problem** is a question for solution.

17. A **Proof** of a solution is the process of testing its correctness.

18. A **Rule** is a concise statement of the method of solving a problem.

19. A **Principle** is a general or settled truth.

Principle of Numeration and Notation.

20. *Ten units of any order are equal to one unit of the next higher order.*

WRITTEN EXERCISES.

21. Copy and read—

1. *100.*	8. *200.*	15. *323.*	22. *982.*
2. *101.*	9. *202.*	16. *175.*	23. *600.*
3. *116.*	10. *220.*	17. *333.*	24. *802.*
4. *126.*	11. *118.*	18. *400.*	25. *111.*
5. *161.*	12. *221.*	19. *404.*	26. *787.*
6. *162.*	13. *212.*	20. *444.*	27. *921.*
7. *170.*	14. *300.*	21. *567.*	28. *129.*

22. Write in figures—

1. Sixty-two.	7. One hundred three.
2. Seventy-five.	8. Two hundred fifteen.
3. Ninety.	9. Three hundred sixty.
4. Fifty-five.	10. Two hundred twenty-one.
5. Eighty.	11. Four hundred forty.
6. Eighty-eight.	12. Five hundred ninety.

23.—1. Copy and read 316405.

SOLUTION. — Separating the given figures into classes or periods of three figures each, we have 316,405. The first period at the left of the decimal point expresses 405 *units*, and the second period 316 *thousands*. The whole is read: three hundred sixteen thousand four hundred five.

Copy and read—

2. *314670.*	8. *101452.*	14. *99000.*
3. *65809.*	9. *39000.*	15. *131311.*
4. *132735.*	10. *7888.*	16. *46006.*
5. *3257.*	11. *50005.*	17. *800000.*
6. *625407.*	12. *683452.*	18. *7314.*
7. *700314.*	13. *700000.*	19. *660066.*

24.—1. Write in figures three hundred sixteen thousand four hundred five.

316,405

SOLUTION.—Writing 316 for the hundreds, tens and ones of *thousands*, and 405 for the hundreds, tens and ones of *units*, gives 316,405, the required expression.

Write in figures—

2. Three hundred seventy-three. Four hundred.
3. Seven hundred sixty-one. Three hundred one.
4. Eight hundred eighty-five. Seventy-nine.
5. One thousand one hundred forty-nine.
6. Nine thousand three hundred thirty-three.
7. Twenty-five thousand three hundred sixty-five.

25. Rule for Numeration.—*Beginning with the lowest order of units, separate the figures of the given number into periods of three figures each.*

In reading, begin with the highest period; read the hundreds, tens and ones of each period; and give the name of each period, except units, after its ones.

26. Rule for Notation.—*Write the figures expressing the hundreds, tens and ones of each period in their order.*

PROBLEMS.

27. Write and read—

1. *711.*	5. *11,224.*	9. *100,000.*	13. *3,163,000.*
2. *309.*	6. *12,560.*	10. *525,125.*	14. *4,725,555.*
3. *1,624.*	7. *50,000.*	11. *500,025.*	15. *65,008,721.*
4. *3,052.*	8. *60,606.*	12. *751,157.*	16. *364,230,000.*

28. Express by figures—

1. Five hundred. Five thousand. Five hundred five.
2. Five thousand five. Eight hundred nine.
3. One thousand one hundred nineteen.
4. Two thousands five hundreds ninety.
5. Eight hundreds seventy-seven.
6. Twenty-five thousands eight hundred eight.
7. Fifty-nine thousands nine hundreds.
8. One hundred sixty-three thousands.

---

29. TEST QUESTIONS.—1. What is a unit? A number? What is a figure?

2. Name the first nine numbers. What numbers are expressed by a single figure? When a number is expressed by two figures, side by side, what does the figure on the right express? The figure on the left?

3. What is numeration? Notation? What does a figure written alone, or at the left of the decimal point, express?

4. How are orders of units expressed? What does the first figure at the left of the decimal point express? The second? The third? Recite the orders of the Numeration Table.

5. What is used to separate figures into classes or periods? What is used to mark the order of primary units?

6. What is a solution? A problem? A proof? A rule? A principle?

7. What is a principle of notation?

SECTION III.

ADDITION.

30.—1. Jane has 5 books, and her sister has 6. How many have both?

2. Albert has 7 white roses and 8 red ones. How many roses has he in all?

3. How many are 5 and 6? How many ones are 8 and 7?

4. How many cents are 8 cents and 4 cents? How many cents are 4 cents and 8 cents?

5. In a garden there are 10 pear trees and 7 peach trees. How many trees are there of the two kinds?

6. A man bought a plow for 10 dollars, and sold it for 2 dollars more than he gave for it. How many dollars did he get for it?

7. A boy rode 8 miles, and walked 7. How far did he travel?

8. In a certain class there are 9 boys and 8 girls. How many pupils are there in the class?

9. How many ones are 10 and 2? 10 and 7? 9 and 8? 10 and 8?

10. How many ones are 9 and 1? 10 and 2? 7 and 4? 9 and 7? 10 and 6?

11. What number is formed by uniting the ones in 6 and 5? 10 and 9?

12. What number is formed by uniting the ones in 7 and 3? 10 and 3?

TABLES.

0 and	1 and	2 and	3 and	4 and
1 are 1	1 are 2	1 are 3	1 are 4	1 are 5
2 " 2	2 " 3	2 " 4	2 " 5	2 " 6
3 " 3	3 " 4	3 " 5	3 " 6	3 " 7
4 " 4	4 " 5	4 " 6	4 " 7	4 " 8
5 " 5	5 " 6	5 " 7	5 " 8	5 " 9
6 " 6	6 " 7	6 " 8	6 " 9	6 " 10
7 " 7	7 " 8	7 " 9	7 " 10	7 " 11
8 " 8	8 " 9	8 " 10	8 " 11	8 " 12
9 " 9	9 " 10	9 " 11	9 " 12	9 " 13
10 " 10	10 " 11	10 " 12	10 " 13	10 " 14

5 and	6 and	7 and	8 and	9 and
1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 " 7	2 " 8	2 " 9	2 " 10	2 " 11
3 " 8	3 " 9	3 " 10	3 " 11	3 " 12
4 " 9	4 " 10	4 " 11	4 " 12	4 " 13
5 " 10	5 " 11	5 " 12	5 " 13	5 " 14
6 " 11	6 " 12	6 " 13	6 " 14	6 " 15
7 " 12	7 " 13	7 " 14	7 " 15	7 " 16
8 " 13	8 " 14	8 " 15	8 " 16	8 " 17
9 " 14	9 " 15	9 " 16	9 " 17	9 " 18
10 " 15	10 " 16	10 " 17	10 " 18	10 " 19

DEFINITIONS.

31. The **Sum** of two or more numbers is a number containing as many units or ones as those numbers.

32. **Addition** is the process of uniting two or more numbers to find their sum.

33. The **Sign of Addition** is an upright cross, +, and is called *plus*.

34. The **Sign of Equality** is =, and is read *equals* or *equal*.

Thus, 9 + 5 = 14, is read *nine plus five equals fourteen*, and means that 5 added to 9 equals 14.

MENTAL EXERCISES.

35.—1. If Henry has 9 dollars and his brother has 7, how many dollars have both?

SOLUTION.—If Henry has 9 dollars and his brother has 7, both have as many dollars as the sum of 9 and 7, which is 16, the answer required.

2. If you gather 7 quarts of berries and your brother 6 quarts, how many do both of you gather?

3. A boy caught 7 fishes, and another boy caught 5. How many did both catch?

4. Harry had 8 cents, and had 5 more given him. How many had he then?

5. How many are 11 and 3? 5 and 8? 10 and 5?

6. John has 6 cents, George 5 and Victor 2. How many cents have they in all?

7. If I give to one boy 5 peaches, to another 7, and keep 4, how many peaches had I at first?

8. How many are 5, 7 and 4? 7, 5 and 4? 5, 4 and 7? 7, 4 and 5? 4, 5 and 7? 4, 7 and 6?

9. How many are 9 and 4? 6, 3 and 2? 3, 6 and 4?

10. How many are 13 and 2? 15 and 2? 15 and 4?

11. What does $11 + 7$ mean? $8 + 3 = 11$?

12. A boy paid 5 cents for an orange, 7 cents for a melon and 6 cents for a pencil. How many cents did he pay for them all?

13. How many are 16 and 2? 16 and 4? 17 and 2? 17 and 3? 18 and 2?

WRITTEN EXERCISES.

36. Copy and add—

(1.)

1
2
3
0
6

SOLUTION.—Begin at the bottom of the column, and say, zero, three, five, six.

Write 6 under the line at the bottom of the column.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
5	4	5	3	1	4	7
7	3	3	2	3	2	0
2	2	2	8	9	0	7
1	1	7	6	7	9	5
15		17		20		19

37. Copy and add—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
8	5	9	4	6	2	9
2	3	0	1	6	1	0
6	0	7	2	2	8	9
4	9	3	4	2	7	0
20		19		16		18

Add each of the above columns downward. If the work is correct, the result will be the same as that first obtained.

MENTAL EXERCISES.

38.—1. How many are 10 and 5? 20 and 5?

2. How many are 30 + 5? 12 + 6? 11 + 7?

3. Count by 2's from 2 to 12.

SOLUTION.—Two, four, six, eight, ten, twelve.

4. Count by 2's from 12 to 30. From 30 to 50.

5. Count by 4's from 4 to 40. From 40 to 80.

6. Count by 5's from 25 to 55. From 30 to 60.

7. Count by 6's from 2 to 32. From 30 to 60.

8. How many are 9 and 2? 19 and 2? 29 and 2? 9 and 3? 19 and 3? 39 and 3?

9. How many are 9 and 4? 29 and 4? 49 and 4? 9 and 5? 39 and 5? 59 and 5? 19 and 5? 39 and 7? 69 and 7?

10. How many are 9 and 6? 49 and 6? 79 and 6? 9 and 8? 19 and 8? 29 and 8? 9 and 9? 29 and 9? 89 and 7?

11. On one branch of a tree are 16 apples, on another 6, and on a third 2. How many apples are there in all?

39.—1. Count by 7's from 3 to 45. From 70 to 98.

2. Count by 8's from 1 to 41. From 64 to 96.

3. How many are 8 and 6? 48 and 6? 78 and 6? 8 and 7? 38 and 7? 58 and 7? 8 and 8? 48 and 8?

4. How many are 8 and 9? 28 and 9? 68 and 9? 8 and 10? 18 and 10? 78 and 10?

5. If a barrel of flour is worth 12 dollars, a hundredweight of fish 5 dollars and a cheese 4 dollars, how many dollars are all worth?

6. What are such numbers as express things of the same kind, as 6 cents and 3 cents, or 7 tens and 4 tens, called? **Similar Numbers.**

7. Can you add 6 cents and 7 tens? Can you add 6 cents and 7 cents?

8. What is the sum of $4 + 3 + 2$? Of $3 + 2 + 4$? Of $2 + 3 + 4$? Of $3 + 4 + 2$? Of $4 + 2 + 3$?

9. How may the correctness of an answer in addition be tested?

By adding the numbers a second time and in a different order.

10. Add 3, 4 and 11, and prove the correctness of the answer.

11. A man gave some apples to 2 boys; to one he gave 4, and to the other 24. If he should give 9 more to each, how many would each have?

Principles of Addition.

40.—1. *Only similar numbers can be added.*

2. *The sum of numbers is the same, in whatever order they are added.*

WRITTEN EXERCISES.

41. Copy, add and prove—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
3	4	5	3	2	9
3	2	7	0	7	3
4	1	2	5	3	4
1	3	0	7	4	5
6	7	9	8	4	7

42. Write, add and prove—

(1.)

16
32
90
138

SOLUTION.—Write the numbers so that the figures expressing ones stand in one column, and the figures expressing tens stand in another column, at the left.

Begin with the ones, and add the ones and tens separately.

Write the sum of the ones under the line at the bottom of the column of ones.

Write the sum of the tens under the line at the bottom of the column of tens.

The result is 138, which is the sum required.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
51	40	44	13	47	91
62	51	31	32	72	15
34	73	41	51	50	12
147					118

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
23	14	42	60	44	17
70	31	85	70	11	61
96	43	10	19	33	50
189					128

43. Write in columns, add and prove—

1. 43, 91, 60 and 14.
2. 50, 75, 11 and 32.
3. 42, 12, 13 and 80.
4. 81, 17, 41 and 70.
5. 55 + 71 + 32 + 20.
6. 63 + 80 + 60 + 17.
7. 16 + 81 + 12 + 50.
8. 60 + 73 + 15 + 31.
9. 43 + 60 + 72 + 19.
10. 50 + 7 + 91 + 10.
11. 92 + 11 + 33 + 12.
12. 80 + 15 + 12 + 71.

MENTAL EXERCISES.

44.—1. James paid 45 cents for a reading-book and 17 cents for a writing-book. How many cents did he pay for both?

SOLUTION.—He paid as many cents as the sum of 45 and 17. 45 and 10 are 55; 55 and 7 are 62: therefore he paid 62 cents.

2. If you should ride 23 miles and walk 13 miles, how far would you travel?

3. How many are 27 + 24? 31 + 9? 61 + 29?

4. George had 30 chickens; he bought 20 more, and had 11 more given him. How many had he then?

5. How many are 7 and 9? 17 and 9? 27 and 9?

6. How many are 16 and 5? 36 and 5? 6 and 6?

7. I bought some beef for 60 cents, some veal for 30 cents, and a fish for 15 cents. How much did the whole cost?

8. Count by 9's from 9 to 63. From 63 to 108.

9. I paid 80 dollars for a cow and 25 dollars for a heifer. How much did both cost?

10. How many are 6 and 7? 16 and 7? 36 and 7?

11. How many are 6 and 9? 26 and 9? 86 and 9? 6 and 10? 36 and 10? 76 and 10?

WRITTEN EXERCISES.

45.—1. Add 855, 345 and 64.

Numbers added,	*855* *345* *64*
Sum,	*1264*

SOLUTION.—Write the numbers so that figures expressing units of the same order stand in the same column.

Begin with the ones, and add the ones, tens and hundreds separately.

The sum of the ones is 14 ones, or 1 ten 4 ones.

Write the 4 ones under the line at the bottom of the column of ones, and add the 1 ten with the tens of the next column.

The sum of the tens is 16 tens, or 1 hundred 6 tens.

Write the 6 tens under the line at the bottom of the column of tens, and add the 1 hundred with the hundreds of the next column.

The sum of the hundreds is 12 hundreds.

Write the 12 hundreds under the line at the bottom of the column of hundreds.

The result is 1264, which is the sum required.

Add and explain in like manner—

(2.)	(3.)	(4.)	(5.)	(6.)
132	*186*	*144*	*191*	*443*
46	*304*	*555*	*49*	*37*
53	*80*	*342*	*723*	*8*

46. Rule for Addition.— *Write the numbers so that all the figures of the same order shall stand in the same column, and draw a line under the columns.*

Begin at the right, add the units of each order separately, and write their sum, if less than ten, under the column added.

If the sum of the units of any order be ten or more, write the figure standing for its ones, and add its tens with the units of the next higher order.

Write the whole sum of the units of the highest order.

PROOF.—Add the numbers a second time, in a different order. The result should be the same by both methods.

PROBLEMS.

47. Find the sum—

1. Of 615, 3045 and 5000.
2. Of 309, 446 and 7131.
3. Of 3241, 445 and 199.
4. Of 27, 366 and 5549.
5. Of 340, 3331 and 19.
6. Of 633, 902 and 1187.

48. Add, explain and prove—

(1.)	(2.)	(3.)	(4.)
134 dollars.	472 men.	160 miles.	341 bushels.
452 "	306 "	49 "	428 "
317 "	28 "	672 "	152 "

(5.)	(6.)	(7.)	(8.)
173 feet.	3142 tons.	9162 bales.	149 horses.
416 "	4152 "	713 "	934 "
712 "	613 "	44 "	196 "

49.—1. What is the sum of 9163 + 120 + 315 + 4?

2. What is the sum of 13162 + 417 + 491 + 82?

3. Add 1345, 7819, 100,000, and 316,271.

(4.)	(5.)	(6.)	(7.)
145 yards.	1000 pounds.	2183 men.	315 cords.
315 "	5006 "	19 "	705 "
410 "	3508 "	94 "	814 "

8. What is the sum of 9168 + 429 + 251 + 128?

9. What is the sum of 3162 + 414 + 490 + 18?

10. Add fifteen thousand three hundred forty-two and nine thousand nine hundred sixty-six.

11. Add one million one thousand nine, five hundred thousand seven hundred eleven and ninety-five thousand four hundred four.

50.—1. Washington was born in the year 1732, and lived 67 years. In what year did he die?

2. A farmer paid 5750 dollars for his farm and 375 dollars for tools. What was the cost of the whole?

3. In one school are 516 pupils, in another 314, and in another 215. How many pupils are there in all?

4. One steamer has on board 1321 bales of cotton, a second 3150 bales and a third 5725 bales. How many bales have the three on board?

5. Smith has 1631 bushels of wheat, Jones 1500 bushels, Woodman 783 bushels, and Kaspar 1735 bushels. How many bushels have they in all?

6. If you should pay 175 dollars for a horse, 225 dollars for a yoke of oxen, 80 dollars for a cart and 93 dollars for a wagon, how much would they all cost you?

51. Write, add and prove—

(1.)	(2.)	(3.)	(4.)	(5.)
12	31	145	123	1412
3	92	54	356	3400
17	16	63	720	570
90	42	20	114	145
43	15	40	333	7682
7	72	68	121	1031
32	13	37	301	915

(6.)	(7.)	(8.)	(9.)	(10.)
25	42	191	333	1614
19	52	37	404	1641
8	71	170	500	901
14	55	45	608	109
62	33	68	171	3170
20	17	22	349	143
9	71	11	100	28

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52. TEST QUESTIONS.—1. What is the sum of two or more numbers? What is addition?

2. What is the sign of addition? What is it called? What is the sign of equality? How is it read?

3. What are such numbers as express things of the same kind called? How may the correctness of an answer in addition be tested?

4. What principles of addition are given?

SECTION IV.

SUBTRACTION.

53.—1. John had 2 apples, and gave 1 of them to his sister. How many had he left?

2. A boy had 3 apples, and sold 2 of them. How many had he left?

3. James had 4 marbles, and lost 2 of them. How many had he left?

4. John has 6 rabbits, and his sister has 3. How many more has John than his sister?

6. If I sold what cost me 5 dollars for 7 dollars, how many dollars more than the cost did I receive?

6. If you have 8 acorns, and should give 5 to your brother, how many would you have left?

7. A hen had 8 chickens, but a hawk took 4 of them. How many chickens were left?

8. A boy is 6 years old; in how many years will he be 10?

9. William is 9 years old, and his brother is 11. What is the difference in their ages?

10. If you have 5 dollars, and should lose 1 dollar, how many dollars would you have left.

11. A merchant bought goods for 9 dollars. How much will he gain by selling them for 11 dollars?

12. How many are 8 less 4? 9 less 7? 11 less 9?

TABLES.

1 from			2 from			3 from			4 from			5 from		
1	leaves	0	2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
2	"	1	3	"	1	4	"	1	5	"	1	6	"	1
3	"	2	4	"	2	5	"	2	6	"	2	7	"	2
4	"	3	5	"	3	6	"	3	7	"	3	8	"	3
5	"	4	6	"	4	7	"	4	8	"	4	9	"	4
6	"	5	7	"	5	8	"	5	9	"	5	10	"	5
7	"	6	8	"	6	9	"	6	10	"	6	11	"	6
8	"	7	9	"	7	10	"	7	11	"	7	12	"	7
9	"	8	10	"	8	11	"	8	12	"	8	13	"	8
10	"	9	11	"	9	12	"	9	13	"	9	14	"	9
11	"	10	12	"	10	13	"	10	14	"	10	15	"	10

6 from			7 from			8 from			9 from			10 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0	10	leaves	0
7	"	1	8	"	1	9	"	1	10	"	1	11	"	1
8	"	2	9	"	2	10	"	2	11	"	2	12	"	2
9	"	3	10	"	3	11	"	3	12	"	3	13	"	3
10	"	4	11	"	4	12	"	4	13	"	4	14	"	4
11	"	5	12	"	5	13	"	5	14	"	5	15	"	5
12	"	6	13	"	6	14	"	6	15	"	6	16	"	6
13	"	7	14	"	7	15	"	7	16	"	7	17	"	7
14	"	8	15	"	8	16	"	8	17	"	8	18	"	8
15	"	9	16	"	9	17	"	9	18	"	9	19	"	9
16	"	10	17	"	10	18	"	10	19	"	10	20	"	10

DEFINITIONS.

54. The **Difference** between two numbers is the number which, when added to the less, will make the greater.

55. **Subtraction** is the process of finding the difference between two numbers.

56. The **Minuend** is the number from which the subtraction is made.

57. The **Subtrahend** is the number which is subtracted.

58. The **Sign of Subtraction** is —, and is called *minus*. When placed between two numbers it denotes that the one on the right of it is to be taken from that on the left.

Thus, 13 — 5 is read, *thirteen minus five*, and means that 5 is to be subtracted from 13.

MENTAL EXERCISES.

59.—1. If a boy had 16 dollars and lost 9 of them, how many had he left?

SOLUTION.—If a boy had 16 dollars, and lost 9 of them, he had left as many as the difference between 16 and 9, which is 7, the answer required.

2. I bought a book for 20 cents and sold it for 12 cents. How many cents did I lose by the sale?

3. William has 14 dollars in one purse and 24 dollars in another. If he should take 5 dollars from each, how many dollars would be left in them?

4. A farmer having 17 dollars, paid 10 dollars for flour. How many dollars had he left? 10 and how many are 17?

5. I paid 19 dollars for a coat and 7 dollars for a

vest. How many dollars more were paid for the coat than for the vest? 7 and how many are 19?

Principles of Subtraction.

60.—1. *Only similar numbers can be subtracted, the one from the other.*

2. *The difference and the subtrahend are together equal to the minuend.*

WRITTEN EXERCISES.

61. Write and subtract—

	(1.)
From	*49*
Take	*26*
Difference,	*23*

SOLUTION.—Write the numbers for subtracting so that the figures representing ones stand in one column, and the figures representing tens stand in another column at the left.

Begin with the ones, and subtract the ones and the tens separately.

Write the difference of the ones, which is 3, under the line at the bottom of the column of ones.

Write the difference of the tens, which is 2, under the line at the bottom of the column of tens.

The result is 23, which is the difference required.

	(2.)	(3.)	(4.)	(5.)	(6)
From	*97*	*82*	*63*	*66*	*97*
Take	*43*	*51*	*43*	*23*	*70*
Difference,	*54*		*20*		*27*

	(7.)	(8.)	(9.)	(10.)
From	*48* books.	*72* yds.	*55* lbs.	*96* dollars.
Take	*13* "	*61* "	*44* "	*16* "

Add the subtrahend and difference in each of the last four problems. Their sum should be equal to the minuend.

MENTAL EXERCISES.

62.—1. How many are 12 less 5? 14 less 8? 18 less 8?

2. How many are 10 less five? 10 less 8? 13 less 9?

3. Count backward by 2's, or by subtracting 2 successively, from 22 to 2.

SOLUTION.—22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2.

4. Count backward by 3's from 60 to 30. From 30 to 3.

5. Count backward by 4's from 48 to 8. From 96 to 72.

6. Count backward by 5's from 40 to 10. From 80 to 55.

7. Count backward by 6's from 90 to 12. From 35 to 11.

8. Count backward by 7's from 61 to 5. From 70 to 35.

9. Count backward by 9's from 45 to 9. From 72 to 45.

10. How many will remain if 2 be taken from 11? 2 from 21? 2 from 31? 2 from 41?

11. How many will remain if 3 be taken from 11? 3 from 21? 3 from 41? 3 from 61?

12. How many will remain if 4 be taken from 11? 4 from 21? 4 from 31? 4 from 51?

63.—1. What number must be added to 7 to make 13? To make 23? To make 33?

2. What number must be added to 6 to make 14?

3. If the subtrahend is 9 and the difference 8, what is the minuend?

4. What number is 19 — 8? 17 — 9? 16 — 7?

5. What number must be added to 8 to make 13? To make 23? To make 33?

6. How many are 21 less 9? 13 less 9?

7. What number must be added to 9 to make 13? To make 23?

8. If you have 9 dollars, how many more must you have to make 16 dollars? To make 26 dollars?

9. If you are now 8 years old, in how many years will you be 14? In how many will you be 24?

10. Jacob has 6 lambs; how many more must he have to make 12? To make 22?

11. 2 tens from 4 tens leave how many tens? 2 tens from 6 tens? 5 tens from 9 tens? 7 tens from 11 tens?

12. A shepherd has 20 sheep in one fold, and 30 in another. How many more has he in the second than in the first?

13. John has 50 cents; how many more must he have to make 70 cents?

WRITTEN EXERCISES.

64.—1. What is the difference between 86 and 47?

	7. 16.
Minuend,	86
Subtrahend,	47
Difference,	39
Proof,	86

SOLUTION.—Write the figures of the subtrahend under the figures of the same order in the minuend.

Since 7 units cannot be taken from 6 units, take 1 ten from the 8 tens of the minuend, leaving 7 tens, and add 10 ones, which are equal to the 1 ten, to the 6 ones, thus making the minuend the same as 7 tens, 16 ones.

Subtract 7 ones from 16 ones, leaving 9 ones, which write under the line at the bottom of the ones.

Subtract 4 tens from 7 tens, leaving 3 tens, which write under the line at the bottom of the tens.

The result is 39, which is the difference required.

Prove the solution by adding the difference to the subtrahend, which gives 86, the minuend.

Subtract, explain in like manner and prove—

	(2.)	(3.)	(4.)	(5.)	(6.)
Minuend,	*93*	*56*	*74*	*87*	*260*
Subtrahend,	*18*	*29*	*65*	*78*	*49*
Difference,	*75*		*9*		*211*

	(7.)	(8.)	(9.)	(10.)	(11.)
Minuend,	*363*	*457*	*554*	*934*	*918*
Subtrahend,	*254*	*148*	*535*	*828*	*909*
Difference,	*109*		*19*		*9*

MENTAL EXERCISES.

65.—1. A farmer had 25 bushels of wheat, and sold 12 bushels. How many bushels had he left?

SOLUTION.—He had left as many bushels as the difference between 25 and 12; $12 = 10 + 2$; 10 from 25 leaves 15, and 2 from 15 leaves 13, which is the answer required.

2. If you had 29 dollars, how many dollars would you have left after spending 16 dollars?

3. What number must be added to 21 to make 40?

4. What number must be added to 13 to make 61?

5. What number must be added to 49 to make 76?

6. Arthur had 28 cents, and his father gave him enough more to make 50. How many cents did his father give him?

7. I had 39 sheep; having sold some, I found there were only 23 left. How many had been sold?

8. Mary's mother is 41 years old; Mary is 13 years old. In how many years will Mary be as old as her mother now is?

9. From a cask containing 63 gallons, 25 gallons leaked out. How many gallons remained in the cask?

10. If you had 47 dollars, and should give away 15, how many dollars would you have remaining?

11. A man sold 31 cattle from a drove of 80. How many remained?

12. In an orchard containing 44 trees, 25 are apple trees, and the rest are pear trees. How many are pear trees?

13. I bought a horse for 90 dollars, and sold him at a loss of 37 dollars. For how many dollars was the horse sold?

14. The cost of a carriage was 100 dollars and that of a harness 43 dollars. How many dollars did the carriage cost more than the harness?

15. A drover bought some cattle for 100 dollars, and sold them for 88 dollars. How many dollars did he lose?

16. If you should exchange a watch worth 75 dollars for a carriage worth 93 dollars, how many dollars would you make by the trade?

17. How much is made by selling a horse which cost 93 dollars, for 105 dollars?

18. I bought a lot of land for 115 dollars, and sold it for 85 dollars. How much did I lose by the transaction?

WRITTEN EXERCISES.

66.—1. From 4334 subtract 2530.

	3. 13.
Minuend,	*4334*
Subtrahend,	*2530*
Difference,	*1804*

SOLUTION.—0 ones from 4 ones leaves 4 ones; 3 tens from 3 tens leaves 0 tens; 5 hundreds cannot be subtracted from 3 hundreds; therefore take 1 thousand from the 4 thousands of the minuend, leaving 3 thousands, and add 10 hundreds, which equal 1 thousand, to the 3 hundreds, thus making the minuend the same as 3 thousands 13 hundreds 3 tens 4 ones; 5 hundreds from 13 hundreds leaves 8 hundreds, and 2 thousands from 3 thousands leaves 1 thousand.

The result is 1804, which is the difference required.

Subtract and explain in like manner—

2. *4562* from *6278*.
3. *835* from *5734*.
4. *2071* from *7665*.
5. *3473* from *3743*.
6. *1205* from *8304*.
7. *600* from *9502*.
8. *1835* from *7035*.
9. *801* from *8010*.

67. Rule for Subtraction.—*Write the subtrahend under the minuend, so that the figures of the same order shall stand in the same column.*

Begin at the right, and subtract the units of each order of the subtrahend from the units of the same order in the minuend, if possible, and write the difference under the order subtracted.

When the units of any order of the minuend are less than those of the same order of the subtrahend, increase their number by adding ten, the value of a unit taken from the next higher order of the minuend, and subtract; then consider the units of that higher order of the minuend one less.

PROOF.—Add the difference and subtrahend together. Their sum should be equal to the minuend.

PROBLEMS.

68. Find the difference between—

1. *1863* and *945*.
2. *1345* and *807*.
3. *87203* and *68315*.
4. *81992* and *85*.
5. *6447* and *5446*.
6. *35702* and *16205*.

69. Perform the subtraction indicated, and explain—

1. *43402 — 1233*.
2. *7206 — 1506*.
3. *8200 — 3415*.
4. *6302 — 4444*.
5. *10931 — 7301*.
6. *14275 — 3900*.
7. *4466 — 2271*.
8. *90043 — 73132*.

70.—1. A man born in the year 1809 died in the year 1870. How many years did he live?

2. A farmer bought a farm for 6390 dollars, and sold it for 9500 dollars. How much did he gain by the transaction?

3. From a cargo of corn containing 1563 bushels, 752 bushels have been taken. How many bushels remain?

4. A certain mountain is 18635 feet high, and another mountain is 15367 feet high. What is the difference in their heights?

5. A merchant sold for 10560 dollars, goods which cost him 8775 dollars. How much did he gain by the transaction?

6. How many is 1000 — 90? 1000 — 11?

7. From nineteen thousands take nineteen.

8. A city has 149306 inhabitants, which is 21319 more than it had five years ago. How many inhabitants had it then?

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71. TEST QUESTIONS.—1. What is the difference between two numbers? What is subtraction? What is the minuend? the subtrahend?

2. What is the sign of subtraction? What principles of subtraction are given?

SECTION V.

REVIEW.

72.—1. By what process do you find the sum of two or more numbers of the same kind?

2. By what process do you find the difference between two numbers of the same kind?

3. How do Addition and Subtraction differ?

4. Can you add, or unite into one number, numbers of different kinds, as 4 feet and 5 dollars?

5. Can you take one number from another of a different kind, as 4 feet from 5 dollars?

6. A man bought a barrel of flour for 17 dollars, some butter for 9 dollars, some molasses for 6 dollars and some sugar for 5 dollars. How many dollars did the whole cost him?

7. In a yard are 26 hens, 6 turkeys and 4 ducks. How many fowls are in the yard? How many would there be left if a hawk should take 7 of them?

8. I bought 28 cows at one time, 7 at another and 6 at another, and afterward sold 9. How many remained?

9. Frank had 45 cents; he spent 11 cents for a writing-book, 8 cents for a pencil and 5 cents for a ball. How many cents had he left?

10. There are 24 hours in a day; if you should spend 10 hours in sleep, 3 in work and 5 in play, how many hours will you have left for study?

11. A company went into battle with 93 men; 20 of them were killed and 10 were taken prisoners. How many were left?

12. George had 50 cents to spend; he bought a ball for 10 cents, a pen for 5 cents, a pencil for 3 cents and a piece of rubber for 8 cents. How many cents had he left?

13. If from 27 you take 15 less 8, how many will remain?

14. A boy bought some articles for 48 cents; for how many cents must he sell them to gain 12 cents? For how many cents to lose 9 cents?

15. Alfred lost 5 cents and found 15 cents, and then had 40 cents. How many cents had he at first?

16. What number must be taken from 39 to leave 10 more than 25?

17. What number must be added to 31 to make a sum 11 less than 51?

18. Susan and Mary have each 17 books; if Susan should give 5 of hers to Mary, how many more would Mary have than Susan?

WRITTEN EXERCISES.

73.—1. What is the sum of three hundred sixty-five, one thousand six hundred fifty, and three hundred twenty-five?

2. If you have 1600 dollars, and should gain 1734 dollars, how much money would you then have?

3. If you have 3334 dollars, and should lose 2500, how much would you have left?

4. The minuend is 11567 and the subtrahend 7457. What is the difference?

5. The subtrahend is 7457 and the difference 4110. What is the minuend?

6. The difference is 4110 and the minuend is 11567. What is the subtrahend?

7. Reed had 1367 tons of coal; he sold at one time 325 tons and at another 572 tons. How many tons had he then left?

572 tons.	*1367* tons.
325 "	*897* "
897 tons.	*470* tons.

SOLUTION.—He sold at one time 325 tons, and at another 572 tons; hence he sold 325 tons + 572 tons, which are 897 tons. Since he had 1367 tons and sold 897 tons, he then had left the difference between 1367 tons and 897 tons, which is 470 tons.

8. I had 3675 dollars, and gained 1320 dollars. If I should pay away 2720 dollars, how much would I have left?

9. How much is 3675 — 539 added to 363 + 73?

10. How much is the sum of 7867 and 1319, diminished by 4261?

SECTION VI.

MULTIPLICATION.

74.—1. If John can remove 2 books at one time, how many can he remove at two times?

2. How many are 2 books and 2 books, or 2 times 2 books?

3. If you have 2 cherries, and James has 3 times as many, how many has James?

4. How many are 2 and 2 and 2, or 3 times 2?

5. If you have 4 fingers on each hand, how many have you on both hands?

6. John has 3 blocks, and his brother has 3 times as many. How many has his brother?

7. If you should get 3 merit-marks each day, how many would you get in 4 days?

8. How many are 4 times 3? 3 times 4?

9. On one twig there are 4 cherries; how many cherries are there on 2 similar twigs?

10. How many cents are 3 times 4 cents?

11. There are four pecks in one bushel; how many pecks are there in 4 bushels?

12. When 6 cents are paid for 1 quart of milk, how much must be paid for 2 quarts?

13. How many are 2 times 6? How many cents are 2 times 6 cents?

14. How many are 5 and 5 and 5, or 3 times 5?

15. At the rate of 5 dollars a week, how many dollars can be earned in 3 weeks?

75.—1. How many are once 1? Once 2? Once 3? Once 4?

2. How many are 2 times 1? 2 times 2? 2 t
3? 2 times 4? 2 times 5?

3. How many are 2 times 6? 2 times 7? 2 ti
8? 2 times 9? 2 times 10?

4. How many are 3 times 1? 3 times 2? 3 times 3? 3 times 4? 3 times 5?

5. How many are 3 times 6? 3 times 7? 3 times 8? 3 times 9? 3 times 10?

6. How many are 4 times 1? 4 times 2? 4 times 3? 4 times 4? 4 times 5?

7. How many are 4 times 6? 4 times 7? 4 times 8? 4 times 9? 4 times 10?

8. How many are 5 times 1? 5 times 2? 5 times 3? 5 times 4? 5 times 5?

9. How many are 5 times 6? 5 times 7? 5 times 8? 5 times 9? 5 times 10?

10. How many are 2 times 2? 3 times 2? 4 times 2? 5 times 2?

11. How many are 6 times 2? 7 times 2? 8 times 2? 9 times 2? 10 times 2?

12. How many are 2 times 3? 3 times 3? 4 times 3? 5 times 3?

13. How many are 6 times 3? 7 times 3? 8 times 3? 9 times 3? 10 times 3?

TABLES.

Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
" 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
" 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
" 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
74 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
m " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

7 times	8 times	9 times	10 times	11 times	12 times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 49	7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 56	8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 63	9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 70	10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 77	11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

DEFINITIONS.

76. **Multiplication** is the process of taking one of two numbers as many times as there are units in the other.

77. The **Multiplicand** is the number to be multiplied.

78. The **Multiplier** is the number by which to multiply.

79. The **Product** is the result of the multiplication.

The multiplicand and multiplier are *Factors* of the product.

80. The **Sign of Multiplication** is an oblique cross, $\times$, and is read *times* or *multiplied by*.

Thus, 9×8 is read *nine multiplied by eight*, or *eight times nine*.

MENTAL EXERCISES.

81.—1. What is the product of 9 by 7? Of 5 by 4?

2. What is the product of 7 by 5? Of 6 by 3?

3. If 1 orange cost 6 cents, how many cents will 8 oranges cost?

SOLUTION.—If 1 orange cost 6 cents, 8 oranges will cost 8 times 6 cents, which are 48 cents.

4. If you can earn 4 dollars in 1 week, how many dollars can you earn in 6 weeks?

5. What will 8 pairs of boots cost at 7 dollars a pair?

6. How many trees in 9 rows of 6 trees each?

7. How many yards are 7 times 10 yards?

8. At 4 dollars each, what will 11 barrels of apples cost?

9. James had 12 chickens, and his brother 6 times as many. How many chickens had his brother?

10. At 7 cents each, what will 9 oranges cost?

11. At 9 cents each, what will 9 pencils cost?

12. If a horse eat 6 quarts of oats in 1 day, how many quarts will he eat in 6 days?

13. At 10 cents each, what will 8 pencils cost?

14. If 11 yards of calico be required for 1 dress, how many yards will be required for 3 dresses?

15. If 12 cents must be paid for a quart of cherries, how many cents must be paid for 9 quarts?

WRITTEN EXERCISES.

82. Copy and multiply—

	(1.)
Multiply	6
By	3
Product,	18

SOLUTION.—Three times six are eighteen. Write 18 under the line at the bottom of the column.

	(2.)	(3.)	(4.)	(5.)	(6.)
Multiply	5	7	8	9	6
By	4	3	8	6	5
Product,	20		64		30

	(7.)
Multiply	23
By	3
Product,	69

Begin with the ones, and multiply the ones and the tens separately.

Write the product of the ones by 3, which is 9, under the column of ones.

Write the product of the tens by 3, which is 6, under the column of tens.

The result is 69, which is the product required.

	(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
Multiply	43	21	31	41	50	60
By	2	4	6	5	7	8
Product,	86		186		350	

MENTAL EXERCISES.

83.—1. What is the product of 9 and 8 multiplied together?

2. What is the product of 3×5? Of $3 \times 5 \times 6$?

3. What is the product of $4 \times 3 \times 2$? Of $3 \times 4 \times 2$? Of $2 \times 3 \times 4$? Of $3 \times 2 \times 4$?

4. Is the product of the factors 4, 3 and 2 the same if multiplied together in different orders?

5. In a certain garden there are 12 rows, with 11 trees in each row. How many trees are there in the garden?

6. James is 12 years old, and his uncle is 5 times as old. What is the age of his uncle?

7. If you earn 10 dollars in 1 month, how much can you earn in 10 months?

8. How many cents must be paid for 12 yards of muslin at 12 cents a yard?

9. How much must be paid for 9 quarts of cherries at 11 cents a quart?

Principle of Multiplication.

84. *The product of factors is the same in whatever order they are multiplied.*

WRITTEN EXERCISES.

85.—1. Multiply 87 by 4.

Multiplicand,	87
Multiplier,	4
Product,	348

SOLUTION.—The product of the 7 ones by 4 is 28 ones, or 2 tens 8 ones.

Write 8 under the line at the bottom of the column of ones, and reserve the 2 tens to unite with the tens of the next product.

The product of the 8 tens by 4 is 32 tens, which, with the 2 tens that were reserved, make 34 tens.

Write 34 under the line at the bottom of the column of tens.

The result is 348, which is the product required.

Write and solve in like manner—

	(2.)	(3.)	(4.)	(5.)	(6.)
Multiply	*93*	*68*	*75*	*34*	*92*
By	*5*	*6*	*8*	*7*	*9*
Product,	*465*		*600*		*828*

	(7.)	(8.)	(9.)	(10.)
Multiply	*125* dollars.	*84* bushels.	*119* barrels.	*137* tons.
By	*3*	*7*	*2*	*5*
Product,	*375* dollars.			*685* tons.

86.—1. Multiply 25 by 10. By 30.

Multiplicand,	*25*
Multiplier,	*10*
Product,	*250*

SOLUTION.—To multiply by 10, simply annex a cipher to the multiplicand, because the annexing of a cipher removes each figure one order to the left, and makes the value denoted tenfold as large as before.

Multiplicand,	*25*
Multiplier,	*30*
Product,	*750*

SOLUTION.—Since 30 is 3×10, the product of 25 by 30 must be the product of $25 \times 3 \times 10$. 3 times 25 is 75, and 10 times 75 is 750.

Write and multiply in like manner—

2. *125* by *10*.
3. *134* by *40*.
4. *133* by *20*.
5. *31* by *100*.
6. *62* by *500*.
7. *124* by *700*.

MENTAL EXERCISES.

87.—1. If it take 6 men 8 days to do a piece of work, how many days will it take 1 man to do it?

SOLUTION.—It will take 1 man 6 times as long as 6 men. If it take 6 men 8 days to do a piece of work, it will take 1 man 6 times 8 days, which are 48 days.

2. If 9 men can mow a field in 3 days, in what time can 1 man mow it?

3. How many days must 1 man work to do a job which would require the labor of 7 men for 5 days?

4. If a certain amount of provisions will last 9 persons 10 days, how long will it last 1 person?

5. George is 8 years old, and his father is 4 years more than 3 times as old. What is the age of his father?

6. If 1 orange be worth 6 apples, how many apples must be given for 9 oranges?

7. If you earn 12 dollars a month, and spend 7 dollars a month, how many dollars can you save in 12 months?

8. When the price of apples is 3 cents each, and of lemons 5 cents each, what will 5 apples and 6 lemons cost?

9. If you can earn 40 cents a day, and you pay 30 cents a day for your board, how many cents can you save in 6 days?

10. If it take 12 men to reap a field in 6 days, how many men will it take to reap it in 1 day?

11. I bought 3 plows at 12 dollars each, and gave in payment 4 ten-dollar bills. How much change ought to be paid back to me?

12. I bought 8 oranges at 5 cents each, and gave in payment a fifty-cent piece. How much change should be paid back to me?

13. How many dollars must be paid for 5 tons of coal at 6 dollars a ton, and 4 cords of wood at 3 dollars a cord?

14. Mary is 10 years old, and her mother is 4 years less than 4 times as old. What is the age of her mother?

15. When blueberries are 9 cents a quart, and blackberries 12 cents, how much more will 8 quarts of blackberries cost than 8 quarts of blueberries?

WRITTEN EXERCISES.

88.—1. Multiply 364 by 42.

Multiplicand,	*364*
Multiplier,	*42*
Partial Products,	*728*
	1456
Product,	*15288*

SOLUTION.—42 is equal to 4 tens and 2 ones. 364 multiplied by 42 must equal 2 times 364 + 4 tens times 364.

Write the multiplier under the multiplicand, placing figures of the same order in the same column.

The product of 364 by the 2 ones is 728 ones, which is a partial product.

4 tens times 4 ones are 16 tens, or 1 hundred + 6 tens; write 6 for the tens of a second partial product, and reserve the 1 hundred to unite with the product of the hundreds; 4 tens times 6 tens are 24 hundreds, which, with 1 hundred reserved, are 25 hundreds, or 2 thousands + 5 hundreds; write 5 for the hundreds of the second partial product, and reserve the 2 thousands to unite with the product of the thousands; 4 tens times 3 hundreds are 12 thousands, which, with 2 thousands, are 14 thousands; write 14 for the thousands of the second partial product, which gives 1456 tens, or 14560.

Find the required product by adding the two partial products, which gives 15288. Prove the work by carefully reviewing it.

Multiply and prove in like manner—

2. *214* by *14*.
3. *509* by *29*.
4. *114* by *16*.
5. *232* by *41*.
6. *444* by *33*.
7. *420* by *17*.

8. *43* by *25*, and prove by reversing the order of the factors.

Multiplicand,	*43*		*25*
Multiplier,	*25*		*43*
Partial Products,	*215*	*Proof*,	*75*
	86		*100*
Product,	*1075*		*1075*

SOLUTION.— Reverse the order of the factors by taking 25 for the multiplicand and 43 for the multiplier.

If the work is correct, the result must be the same by both processes, because the product of any set of factors must be the same in whatever order they may be multiplied.

9. *67* by *39*, and prove by reversing the order of the factors.

89. Rule for Multiplication. — *Write the multiplier under the multiplicand, placing figures of the same order in the same column.*

If the multiplier consists of but one order of units, multiply each order of the multiplicand in succession, beginning at the right, by the multiplier, writing the right-hand figure of the result under the order multiplied, and uniting the units expressed by the left-hand figure, if any, with the next result.

If the multiplier consists of more than one order of units, multiply each order of the multiplicand by each order of the multiplier, write the right-hand figure of each partial product under the

order of the multiplier, and add the partial products.

If either factor have one or more ciphers on the right, multiply without regard to the cipher or ciphers on the right of either factor, and annex to the result as many ciphers as there are on the right of the factors.

PROOF.—Review the work carefully, or reverse the order of the factors and multiply. The result should be the same by both methods.

PROBLEMS.

90. Find the product of—

1. *1251* dollars by *6*.
2. *903* hogsheads by *4*.
3. *1037* yards by *7*.
4. *4009* bushels by *9*.
5. *365* days by *27*.
6. *58* miles by *43*.
7. *731* tons by *1000*.
8. *3140* acres by *120*.

91.—1. What is the product of 642 multiplied by 403?

Multiplicand,	*642*
Multiplier,	*403*
	1926
	2568
Product,	*258726*

Omit to multiply by the 0 tens, because the product of any number by 0 is 0.

2. What is the product of 316 multiplied by 502?
3. What is the product of 1207×2001?
4. How many are 1005 times 1005?
5. What is the product of 56390 multiplied by 401?
6. What will 125 horses cost at 108 dollars each?
7. How many pounds of beef are there in 107 barrels, if each barrel contains 200 pounds?

8. How many bricks in 203 loads, if each load contains 1037 bricks?

92.—1. What will a farm of 164 acres cost at 25 dollars per acre?

2. How many soldiers in 16 regiments, if each regiment contains 819 men?

3. If sound travel 1120 feet in 1 second, how many feet will it travel in 105 seconds?

4. The multiplier is 68 and the multiplicand is 4320. What is the product?

5. What is the product of one hundred thirteen multiplied by itself?

6. How many cents will 3184 bushels of corn cost at 87 cents a bushel?

7. What is the product of $555 \times 44 \times 6$?

8. In a certain storehouse there are 21 apartments, and in each apartment 15 bins, each bin containing 490 bushels of wheat. How many bushels of wheat are there in all?

9. A merchant shipped 11109 boxes of sugar, each weighing 198 pounds. What was the weight of the whole?

10. If light travel at the rate of 192,000 miles in a second, how far will it travel in 5 minutes of 60 seconds each?

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93. TEST QUESTIONS.—1. What is multiplication? The multiplicand? The multiplier? The product?

2. What is the sign of multiplication? How is it read? How does it differ in form from the sign of addition?

3. What are the factors of a product? Name a principle of multiplication.

SECTION VII.

DIVISION.

94.—1. Two cows have 4 horns. How many horns has each cow?

2. How many times 2 horns are 4 horns?

3. A farmer has 9 sheep. How many times 3 sheep has he?

4. Nine sheep are how many times 3 sheep?

5. A harrow has 12 teeth. How many times 3 teeth has it? How many times 4 teeth has it?

6. Twelve is how many times 4? How many times 3?

7. Eight fowls are how many times 2 fowls?

8. A house has 16 windows. How many times 4 windows has it?

95.—1. How many times 1 in 2? In 3? In 4? In 5? In 6?

2. How many times 2 in 2? In 4? In 5? In 6? In 8? In 10?

3. How many times 2 in 12? In 14? In 16? In 18? In 20?

4. How many times 3 in 3? In 6? In 9? In 12? In 15?

5. How many times 3 in 18? In 21? In 24? In 27? In 30?

6. How many times 4 in 4? In 8? In 12? In 16? In 20?

7. How many times 4 in 24? In 28? In 32? In 36? In 40?

8. How many times 5 in 5? In 10? In 15? In 20? In 25?

9. How many times 5 in 30? In 35? In 40? In 45? In 50? In 60?

10. What is one of the 2 equal parts of 6? Of 10? Of 16?

11. What is one of the 3 equal parts of 12? Of 15? Of 24?

12. What is one of the 4 equal parts of 8? Of 16? Of 40?

13. What is one of the 5 equal parts of 10? Of 20? Of 35? Of 45?

14. What is one of the 6 equal parts of 6? Of 12? Of 24? Of 30?

15. What is one of the 7 equal parts of 7? Of 21? Of 35? Of 42?

16. What is one of the 8 equal parts of 8? Of 32?

TABLES.

1 in	2 in	3 in	4 in	5 in
1, Once.	2, Once.	3, Once.	4, Once.	5, Once.
2, 2 times.	4, 2 times.	6, 2 times.	8, 2 times.	10, 2 times.
3, 3 "	6, 3 "	9, 3 "	12, 3 "	15, 3 "
4, 4 "	8, 4 "	12, 4 "	16, 4 "	20, 4 "
5, 5 "	10, 5 "	15, 5 "	20, 5 "	25, 5 "
6, 6 "	12, 6 "	18, 6 "	24, 6 "	30, 6 "
7, 7 "	14, 7 "	21, 7 "	28, 7 "	35, 7 "
8, 8 "	16, 8 "	24, 8 "	32, 8 "	40, 8 "
9, 9 "	18, 9 "	27, 9 "	36, 9 "	45, 9 "
10, 10 "	20, 10 "	30, 10 "	40, 10 "	50, 10 "

6 in	7 in	8 in	9 in	10 in
6, Once.	7, Once.	8, Once.	9, Once.	10, Once.
12, 2 times.	14, 2 times.	16, 2 times.	18, 2 times.	20, 2 times.
18, 3 "	21, 3 "	24, 3 "	27, 3 "	30, 3 "
24, 4 "	28, 4 "	32, 4 "	36, 4 "	40, 4 "
30, 5 "	35, 5 "	40, 5 "	45, 5 "	50, 5 "
36, 6 "	42, 6 "	48, 6 "	54, 6 "	60, 6 "
42, 7 "	49, 7 "	56, 7 "	63, 7 "	70, 7 "
48, 8 "	56, 8 "	64, 8 "	72, 8 "	80, 8 "
54, 9 "	63, 9 "	72, 9 "	81, 9 "	90, 9 "
60, 10 "	70, 10 "	80, 10 "	90, 10 "	100, 10 "

96.—1. If any number of things are separated into two equal parts, what is each part called?

One half of the number of things.

2. If Arthur has 6 apples, and should distribute them equally between 2 friends, how many apples would each friend receive?

SOLUTION.—If Arthur has 6 apples, and should distribute them equally between 2 friends, each friend would receive one half of 6 apples, which is 3 apples.

3. If you should wish to distribute 10 cents equally

between 2 boys, what part of 10 cents would each boy receive?

4. If you have 12 pears, and should give one half of them to your brother, how many would he receive?

5. If any number of things be separated into three equal parts, what is each part called?

One third of the number of things.

6. If 3 boys should share 12 apples, equally dividing them, what part of 12 apples would each have?

7. What is one third of 18 dollars? Of 24 gallons? Of 30 bushels? Of 36 days?

8. When any number is separated into four equal parts, what is each part called?

One fourth of that number.

9. If 20 pears be shared equally among 4 boys, what part of the 20 pears will each boy have?

10. How many is one fourth of 16? Of 24?

11. If any number be separated into five equal parts, what is each of the parts called?

One fifth of the number.

12. How many is one fifth of 35? Of 45? Of 50?

13. When any number is separated into six equal parts, into seven equal parts, etc., what is each of these parts called?

One sixth, one seventh, etc., of the number.

14. What is one sixth of 48? Of 60? One seventh of 21? Of 35? One eighth of 24? Of 40? Of 72?

15. If 6 hats cost 48 dollars, what will 1 hat cost?

16. What is one tenth of 30? Of 100? Of 120?

17. If 10 barrels of flour cost 100 dollars, what will 1 barrel cost?

DEFINITIONS.

97.—1. **Division** is the process of finding how many times one of two given numbers is contained in the other; or,

Division is the process of separating one of two given numbers into as many equal parts as there are units in the other.

98. The **Dividend** is the number to be divided.

99. The **Divisor** is the number by which to divide.

100. The **Quotient** is the result of the division.

101. The **Sign of Division** is a short horizontal line, with a dot above and another below it, ÷, or a short upright curved line,), and is read *divided by.*

Thus, 40 ÷ 8, or 8)40, is read, *forty divided by eight.*

WRITTEN EXERCISES.

102. Copy and divide-

(1.)

Divisor, 2)8 *Dividend.*

4 *Quotient.*

SOLUTION.—2 is contained in 8, 4 times. Write 4 under the dividend for the quotient.

(2.) 3)18 (3.) 5)15 (4.) 4)12 (5.) 6)30 (6.) 8)48

(7.)

8)50

6, 2 *Rem.*

SOLUTION.—8 is contained in 50, 6 times, and 2 remain. Write 6 as the quotient, and the 2 as a remainder.

(8.) 7)57 (9.) 6)55 (10.) 8)75 (11.) 9)47 (12.) 7)58

Multiply the quotient by the divisor, and to this

product add the remainder, in each of the last five problems. If this result equals the dividend, the work is correct.

MENTAL EXERCISES.

103.—1. What do you understand by one half of a number?

One of the two equal parts into which the number is divided.

2. What do you understand by one third of a number? By one fourth of a number?

3. What do you understand by two thirds of a number? By two fourths of a number?

4. What part of 5 is 1? Is 2? Is 3? Is 4?

SOLUTION.—One is 1 fifth of 5; 2 is 2 times 1 fifth of 5, or 2 fifths of 5; 3 is 3 times 1 fifth of 5, or 3 fifths of 5; and 4 is 4 times 1 fifth of 5, or 4 fifths of 5.

5. What part of 6 is 1? Is 2? Is 3? Is 4? Is 5?

6. Seven is how many times 3?

SOLUTION.—Seven is 2 times 3, and 1 remains, which is 1 third of 3. Therefore, 7 is 2 and 1 third times 3.

7. Nine is how many times 2? 4? 3? 8?

8. Eight is how many times 6?

9. Nine is how many times 6? 7? 8?

10. At 8 dollars a yard, how many yards of velvet can be bought for 33 dollars?

11. At 9 cents a pound, how many pounds of rice can be bought for 47 cents?

12. 15 is how many times 4?

SOLUTION.—Fifteen is 3 times 4, with a remainder 3; or 3 and 3 fourths times 4.

13. How many yards of braid, at 7 cents a yard, can be bought for 52 cents?

14. Thirty-eight is how many times 7? 5?

15. Fifty-nine is how many times 5? 8?

16. If 9 pieces of cloth cost 95 dollars, what is the cost of each piece?

17. Eighty-three is how many times 9? 7?

18. What is one fifth of 52? Of 63? Of 49?

19. What is one seventh of 31? Of 57? Of 68?

20. What is one eighth of 46? Of 39? Of 73?

21. If 75 persons are to cross a stream in a boat which will carry only 8 persons at a time, how many persons will remain after as many full boat-loads as possible have crossed?

22. If 10 bushels will fill a bin, how many bins can be filled from 87 bushels of wheat, and how many bushels will remain?

23. How many plows, worth 11 dollars each, can be bought with 93 dollars, and how many dollars will be left?

24. In one hundred and thirty eggs are how many dozen, and how many remain?

DEFINITIONS.

104. A **Remainder** is a part of the dividend which may remain undivided.

105. An **Integer** is a number composed of entire units or ones.

106. A **Fraction,** as one half, two thirds, etc., is a number which represents one or more of the equal parts of a unit or one.

One half is written $\frac{1}{2}$, one third is written $\frac{1}{3}$, two thirds are written $\frac{2}{3}$, three fourths are written $\frac{3}{4}$, and so on. In such expressions the number written below the line denotes the number of parts into which the unit is divided, and the number written above the line denotes the number of parts taken.

Principles of Division.

107.—1. *Division is the reverse of multiplication.*

2. *The dividend is equal to the product of the integer of the quotient multiplied by the divisor, plus the remainder.*

WRITTEN EXERCISES.

108.—1. Divide 4315 by 4, or separate 4315 into 4 equal parts; and prove the solution.

Divisor, $4)\overline{4315}$ *Dividend.*

$1078\frac{3}{4}$ *Quotient.*

SOLUTION.—One of the 4 equal parts of a number is one fourth of that number.

One fourth of 4 thousands is 1 thousand. Write 1 in the thousands' order in the quotient.

One fourth of 3 hundreds is 0 number of hundreds. Write 0 in the hundreds' order in the quotient, and unite the 3 hundreds with the 1 ten, making 31 tens.

One fourth of 31 tens is 7 tens, with 3 tens remaining. Write 7 in the tens' order in the quotient, and unite the 3 tens with the 5 ones, making 35 ones.

One fourth of 35 ones is 8 ones, with 3 ones remaining. Write the 8 in the ones' order in the quotient.

Write the remainder, 3, over the divisor as a part of the quotient.

The 3 written over 4, or $\frac{3}{4}$, may be regarded as indicating the division of 3 by 4.

The result is $1078\frac{3}{4}$, which is the quotient required.

1078
4
4312
3
4315

PROOF.—To prove the work, multiply the integer of the quotient by the divisor, and add to the product the remainder. If the work is correct, this result will equal the dividend.

When only the divisor, dividend and quotient are written, the process is called **Short Division.**

Divide, by short division, explain and prove—

(2.)	(3.)	(4.)	(5.)
$4)3101$	$5)5163$	$7)814$	$8)1137$
$775\frac{1}{4}$	$1032\frac{3}{5}$		$142\frac{1}{8}$

(6.)	(7.)	(8.)	(9.)
$7)300$	$3)975$	$6)1992$	$9)2889$

10. How many is one fourth of 731 apples?
11. What is the quotient of 1363 days divided by 2?
12. What is the quotient of 1563 divided by 10?

10)1563
156.3
Or, $156\frac{3}{10}$

SOLUTION.—To divide by 10, simply move the decimal point in the dividend one order to the left, because this changes each figure to an order next lower, and makes the value expressed only one tenth as large as before.

The remainder is 3, and written at the right of the point, expresses 3 tenths, or written over the divisor, $\frac{3}{10}$.

The result is $156\frac{3}{10}$, which is the quotient required.

13. What is the quotient of 715 divided by 100?
14. What is the quotient of 1634 divided by 10?
15. What is the quotient of 1783 divided by 1000?

MENTAL EXERCISES.

109.—1. How many pencils, at 7 cents each, can be bought for 56 cents?

SOLUTION.—Since one pencil can be bought for 7 cents, as many pencils can be bought for 56 cents as there are times 7 in 56, which are 8 times.

Therefore, 8 pencils at 7 cents each can be bought for 56 cents.

2. How many barrels of flour, at 8 dollars each, can be bought for 64 dollars?

3. How many pounds of sugar, at 9 cents a pound, can be bought for 72 cents?

4. If you have 60 dollars, and spend it at the rate of 10 dollars a week, how many weeks will your money last you?

5. If twelve eggs are 1 dozen, how many dozen are 72 eggs?

6. When pine-apples are 8 cents each, how many can be bought for 64 cents?

7. When 96 dollars are paid for 12 barrels of flour, how much is paid for each barrel?

8. How many melons can be exchanged for 110 peaches, at the rate of 11 peaches for 1 melon?

9. How many pounds of beef, at 11 cents a pound, can be bought for 79 cents, and how many cents will remain?

10. At 12 cents a pound, how many pounds of sugar, and what part of a pound, can be bought for 109 dollars?

11. How many are 12 times 9, plus 1? 9 times 12, plus 1?

12. John gained 24 dollars by purchasing wood at 5 dollars a cord and selling it at 8 dollars a cord. How many cords did he buy?

WRITTEN EXERCISES.

110.—1. Divide 672 by 4, and prove the solution.

Divisor. Divid. Quot.

$$4)672(168$$
$$\underline{4}$$
$$27$$
$$\underline{24}$$
$$32$$
$$\underline{32}$$

SOLUTION.—4 is contained in 6 hundreds, 1 hundred times. Write 1 in the order of hundreds in the quotient, which for convenience is placed on the right of the dividend.

Multiply 4, the divisor, by the 1 hundred, making 4 hundreds, which write under the 6 hundreds, and subtract from it, leaving 2 hundreds. Unite the 2 hundreds with the 7 tens, making 27 tens.

4 is contained in 27 tens, 6 tens times. Write the 6 in the order of tens in the quotient.

Multiply the divisor by the 6 tens, making 24 tens, which write under the 27 tens, and subtract, leaving 3 tens. Unite the 3 tens with the 2 ones, making 32 ones.

4 is contained in 32 ones, 8 times. Write the 8 in the order of ones in the quotient. The result is 168, which is the quotient required.

$$168$$
$$\underline{4}$$
$$672$$

PROOF.—To prove the work, multiply the quotient by the divisor. If this product equals the dividend, the work is correct.

When each step of the solution is written, the process is called **Long Division.**

Divide and prove in like manner—

2. *540* by *5*.	5. *423* by *3*.	8. *936* by *8*.
3. *896* by *2*.	6. *572* by *4*.	9. *796* by *2*.
4. *822* by *6*.	7. *655* by *5*.	10. *504* by *7*.

111. Rule for Division.—*Write the divisor at the left of the dividend.*

Divide the least number of the left-hand orders

of the dividend that will contain the divisor, and place the quotient at the right of the dividend, in long division, and beneath the dividend in short division.

Multiply the divisor by this quotient; subtract the result from that part of the dividend which was used; to the remainder annex the next order of the dividend, and divide the number thus formed.

Proceed in the same manner until all the orders of the dividend have been used.

Should there be at last a remainder, write it, with the divisor under it, as a part of the quotient.

When the divisor is 1, with one or more ciphers on the right, move the decimal point in the dividend as many orders to the left as there are ciphers on the right of the divisor. The orders on the left of the point will be the integer of the quotient, and the orders on the right the remainder expressed as a fractional part of the quotient.

PROOF.—Multiply the integer of the quotient by the divisor, and add to the product the remainder, if any. If the work is correct, this result will equal the dividend.

PROBLEMS.

112. Divide, explain and prove—

(1.)	(2.)	(3.)	(4.)
2)1363	*8)4602*	*7)703*	*4)2060*
(5.)	(6.)	(7.)	(8.)
14)4651	*15)3910*	*21)443*	*12)6702*

113. How many is—

1. One fourth of 731 apples?
2. One third of 563 cents?
3. One sixth of 802 feet?
4. One seventh of 415 rods?
5. One ninth of 629 dollars?

What is the quotient of—

6. 6363 days ÷ 11?
7. 2741 pounds ÷ 13?
8. 1790 dollars ÷ 7?
9. 8000 cents ÷ 25?
10. 4350 yards ÷ 9?

114.—1. What is one tenth of 3587?

2. If 15 acres of land cost 6090 dollars, how many dollars will one acre cost?

3. How many hours will it take a train of cars to move 1225 miles, at the rate of 25 miles per hour?

4. What number is equal to 1488 ÷ 24?

5. What number is equal to 4141 ÷ 101?

6. How many times may a 31-gallon cask be filled from a vat containing 1929 gallons, and how many gallons will remain?

7. Into how many lots of 10 acres each can 365 acres of land be divided, and how many acres will remain undivided?

8. If 100 feet of boards will make a box, how many similar boxes can be made from 5767 feet of boards, and how many feet will remain?

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115. TEST QUESTIONS.—1. What is division? What is the dividend? The divisor? The quotient?

2. What is the sign of division? Show how it is used? What does it mean?

3. What is a remainder? An integer? A fraction?

4. What is short division? Long division? What principles of division are given?

SECTION VIII.

REVIEW.

116.—1. By what process do you find the product of two or more numbers?

2. By what process do you find the quotient of one number divided by another?

3. How do multiplication and division differ?

4. If the multiplicand is dollars, what will be the denomination of the product?

5. If the multiplicand is 6 and the multiplier 4, how many times the multiplicand is the product?

6. What does the product express?

7. If the divisor is 4 dollars and the dividend 24 dollars, what does the quotient express?

8. When the divisor is 4 and the dividend 24 dollars, what part of 24 dollars is the quotient?

9. To what term in multiplication does the dividend in division correspond?

10. To what terms in division do the factors in multiplication correspond?

11. How many are 7 times 12, plus 8?

12. How many are 9 times 11, minus 7?

13. When 5 tons of coal cost 35 dollars, what will 7 tons cost?

SOLUTION.—If 5 tons cost 35 dollars, 1 ton will cost one-fifth of 35 dollars, or 7 dollars, and 7 tons will cost 7 times 7 dollars, which are 49 dollars.

14. When 8 barrels of flour cost 88 dollars, what will 5 barrels cost?

15. James bought 12 bags of meal for 24 dollars, and Smith bought 11 bags at the same rate. How many dollars did Smith pay for his meal?

16. If you can earn 56 dollars in 8 weeks, how much can you earn in 3 weeks?

17. If a horse can trot at the rate of 24 miles in 3 hours, how far can he trot in 2 hours?

18. If 5 men can do a piece of work in 6 days, in how many days can 3 men do it?

SOLUTION.—If 5 men can do a piece of work in 6 days, 1 man will require 5 times 6 days, or 30 days, to do it, and 3 men will require one third of 30 days, or 10 days.

19. If 6 men can mow a field in 8 days, in how many days can 12 men mow it?

20. When 8 bushels of wheat will pay for 4 yards of cloth at 4 dollars a yard, what is the value of a bushel of wheat?

21. When 4 yards of cloth can be bought for 16 dollars, how many yards of cloth can be bought for 40 dollars?

SOLUTION.—If 4 yards can be bought for 16 dollars, 1 yard can be bought for one fourth of 16 dollars, or 4 dollars. Since 1 yard can be bought for 4 dollars, as many yards can be bought for 40 dollars as there are times 4 in 40, or 10.

22. If 5 horses will consume 60 bushels of oats in a certain time, how many horses will consume 72 bushels in the same time?

23. When 99 dollars will pay for 11 barrels of flour, how many barrels can be bought for 63 dollars?

24. If you can earn 108 dollars in 12 months, how many dollars can you earn in 8 months?

WRITTEN EXERCISES.

117.—1. A merchant bought 12 hogsheads of molasses at 65 dollars a hogshead, and sold them for 805 dollars. How much did he gain by the transaction?

2. From 6304×15, subtract $372 \div 3$.

3. If you sell 506 hats, which cost you 4 dollars each, for 6 dollars each, how much will you gain?

4. From 1035×6, take 1060×5.

5. I bought 8 horses at 225 dollars each, and 17 yoke of oxen at 150 dollars a yoke. What is the cost of the whole?

6. If a man's salary is 5000 dollars a year, and his expenses are 225 dollars a month, how much can he save each month?

7. A farmer has 131 bushels of corn, twice as much rye, and three times as much wheat. What quantity of rye and of wheat has he?

8. When 19 acres of land cost 760 dollars, how many acres can be bought for 2180 dollars?

9. If a quantity of provisions will last 355 men 25 days, how long will it last 5 men?

10. When 755 dollars are paid for 5 horses, how much must be paid for 31 horses?

11. I bought a farm containing 120 acres for 7200 dollars, and sold it for 75 dollars an acre. How much did I gain by the transaction?

12. How much is 255×12 divided by 51×8?

13. How many casks, holding 31 gallons each, can be filled from 17 hogsheads of molasses, containing each 93 gallons?

SECTION IX.

FACTORS.

118.—1. How many ones in 2? In 3? In 10?

2. What two integers, then, multiplied together, will produce 2? 3? 10? What 15? 21?

3. What integers, other than itself and 1, multiplied together, will produce 14? 22? 39?

4. What integers, other than itself and 1, will divide 14 without a remainder? 22? 39?

5. What integers, multiplied together, will produce 9? 10? 16? 18?

6. What are all the integers which will divide 9 without a remainder? 10? 16? 18?

7. Name some number which is not the product of any integers except itself and 1.

8. Name some number which is the product of other integers than itself and 1.

DEFINITIONS.

119. The **Factors** of a number are the integers which, being multiplied together, will produce that number.

120. A **Prime Number** is an integer that has no factor except itself and 1.

121. A **Composite Number** is an integer that has other factors besides itself and 1.

122. The **Prime Factors** of a number are the prime numbers that are factors of that number.

Since 1 is a factor of all numbers, in naming the prime factors of numbers it need not be given.

123. **Factoring** is the process of finding the factors of composite numbers.

124.—1. What are the prime factors of 12? 14? 16?

2. What are the prime factors of 15? 18? 20?

3. What are the prime factors of 21? 28? 30?

4. What are the prime factors of 33? 45? 49?

5. What are the prime factors of 27? 42? 44?

6. What are the prime factors of 36? 40? 54?

7. Name the prime numbers from 1 to 19. From 19 to 29.

8. Name the prime numbers from 31 to 41. From 41 to 83.

9. Name the composite numbers from 4 to 20. From 24 to 36. From 36 to 48. From 48 to 60.

125. Principle.—*Every composite number is equal to the product of all its prime factors.*

WRITTEN EXERCISES.

126.—1. What are the prime factors of 90?

$2)\overline{90}$
$3)\overline{45}$
$3)\overline{15}$
5

$90 = 2 \times 3 \times 3 \times 5$

SOLUTION.—Dividing 90 by the prime number 2, we have as factors 2 and the composite number 45.

Dividing 45 by the prime number 3, we have as factors 3 and 15. Dividing again by 3, we have as factors 3 and 5, which are prime numbers. Hence, $90 = 2 \times 3 \times 3 \times 5$, and the prime factors of 90 are 2, 3, 3 and 5.

2. What are the prime factors of 84?

3. What are the prime factors of 75?

4. What are the prime factors of 96?

127. Rule for finding the Prime Factors of a Number.—*Divide the given number by any prime number greater than 1 that will divide it without a remainder, and the quotient, if composite, in the same manner; and so proceed until a quotient is obtained which is a prime number.*

The last quotient and the several divisors are the prime factors.

PROBLEMS.

128. What are the prime factors of—

1. 95?	4. 122?	7. 108?	10. 148?
2. 63?	5. 116?	8. 200?	11. 210?
3. 92?	6. 184?	9. 728?	12. 735?

SECTION X.

DIVISORS.

129.—1. What integers will divide 15 without a remainder? 21? 35?

2. What integers will divide 16 without a remainder? 27? 42?

3. What integers will divide 44 without a remainder? 77? 81?

4. What factors have 6 and 24 in common? 15 and 20?

5. What factors have 22 and 25 in common?

6. What prime factors have 12 and 18 in common?

7. What is the greatest factor common to 12 and 18?

8. What is the product of the prime factors common to 12 and 18?

9. What is the product of the prime factors common to 8 and 24?

10. What is the greatest factor common to 8 and 24?

DEFINITIONS.

130. A **Divisor,** or **Measure,** of a number is any factor of that number.

131. A **Common Divisor** of two or more numbers is any factor common to those numbers.

132. The **Greatest Common Divisor** of two or more numbers is the greatest factor common to those numbers.

133. Principle.—*The greatest common factor, or divisor, of two or more numbers is equal to the product of all the common prime factors of those numbers.*

WRITTEN EXERCISES.

134.—1. What is the greatest common factor or divisor of 330 and 550?

$330 = 2 \times 3 \times 5 \times 11$
$550 = 2 \times 5 \times 5 \times 11$
$2 \times 5 \times 11 = 110$

SOLUTION.—Find the prime factors of the numbers.

The prime factors common to the numbers are 2, 5 and 11, and their product is 110. Hence, the greatest common factor or divisor is 110.

2. Find the greatest common factor of 27 and 36.

3. Find the greatest common factor of 42 and 35.

4. Find the greatest common divisor of 14, 35 and 63.

135. Rule for finding the Greatest Common Divisor.—*Find the prime factors common to the given numbers, and the product of those factors will be the greatest common divisor of the numbers.*

PROBLEMS.

136. What is the greatest common factor or divisor of—

1. 26 and 39?	4. 18 and 96?	7. 4, 6 and 18?
2. 17 and 51?	5. 45 and 300?	8. 12, 30 and 84?
3. 27 and 63?	6. 21 and 105?	9. 18, 54 and 72?

10. What is the length of the longest pole which will exactly measure 130, 150 or 170 feet?

SECTION XI.

MULTIPLES.

137.—1. Name the numbers from 3 to 15 that will contain 3 without a remainder.

2. What are some of the numbers that are an exact number of times 3?

3. What are some of the numbers that are an exact number of times 5? Of times 8? Of times 9?

4. Of what integers is 12 an exact number of times?

5. Name some number that will contain either 3 or 4 an exact number of times.

6. Name some dividend that will exactly contain 2 and 6. 5 and 7. 6 and 9.

7. What is the least dividend that will contain both 10 and 15 an exact number of times?

8. What are all the prime factors of 10 and 15?

9. What are prime factors of the least number that will exactly contain both 10 and 15?

DEFINITIONS.

138. A **Multiple** of a number is any number which it will divide without a remainder.

139. A **Common Multiple** of two or more numbers is any number which each of those numbers will divide without a remainder.

140. The **Least Common Multiple** of two or more numbers is the least number that each of those numbers will divide without a remainder.

141. Principle.—*The least common multiple of two or more numbers is the least number that contains all the prime factors of those numbers.*

WRITTEN EXERCISES.

142.—1. What is the least common multiple of 4, 12 and 30?

$4 = 2 \times 2.$
$12 = 2 \times 2 \times 3.$
$30 = 2 \times 3 \times 5.$
$2 \times 2 \times 3 \times 5 = 60.$

SOLUTION.—Find the prime factors of the given numbers.

The multiple of 4 must contain the prime factors 2 and 2; the multiple of 12, the additional prime factor 3; and the multiple of 30, the additional prime factor 5. These factors are 2, 2, 3 and 5, and their product is 60. Therefore, the least common multiple of 4, 12 and 30 is 60.

2. Find the least common multiple of 8 and 12.

3. Find the least common multiple of 4, 6 and 20.

143. Rule for finding the Least Common Multiple.—*First find the prime factors of the given numbers. The product of these different prime factors, each factor being taken the greatest number of times it occurs in any of the numbers, will be their least common multiple.*

PROBLEMS.

144. What is the least common multiple of—

1. 16 and 18?	4. 14 and 21?	7. 6, 8 and 10?
2. 36 and 108?	5. 42 and 63?	8. 12, 15 and 16?
3. 24 and 84?	6. 19 and 57?	9. 16, 35 and 70?

10. What is the smallest sum of money for which you can purchase a number of pears at either 5 cents, 10 cents or 12 cents each?

SECTION XII.

CANCELLATION.

145.—1. What is the quotient of 60 divided by 15?

2. What is the quotient of one-third of 60 divided by one-third of 15?

3. What is the quotient of one-fifth of 60 divided by one-fifth of 15?

4. What common factors have 60 and 15?

5. If you take out from 60 the factors common to 15, what factor will remain?

6. What is the quotient of $2 \times 7 \times 3$ divided by 2×7?

7. If you strike out from 42 and 14 all common factors, and divide the remaining factors, what is the quotient?

DEFINITION.

146. **Cancellation** is the process of shortening computations by striking out equal factors from the dividend and divisor, and using only the remaining factors.

147. Principle.—*Striking out equal factors from both dividend and divisor, or dividing them by the same number, does not change their quotient.*

WRITTEN EXERCISES.

148.—1. Divide $11 \times 3 \times 2$ by 11×2.

$$\frac{\cancel{11} \times 3 \times \cancel{2}}{\cancel{11} \times \cancel{2}} = \frac{1 \times 3 \times 1}{1 \times 1} = 3$$

SOLUTION.—Write the dividend over the divisor. Divide both dividend and divisor by 11 and 2, by canceling or striking out those common factors in both, leaving $1 \times 3 \times 1 \div 1 \times 1 = 3$. When a factor is canceled, 1 remains, and if not written is understood.

2. Divide 150 by 30.

$$3\,|\,0)15\,|\,0 \quad 5$$

SOLUTION.—Since the dividend and divisor have 10 as a common factor, cancel it by striking off the cipher from the right of both, leaving 3)15, which equals 5.

3. Divide $13 \times 11 \times 7$ by 13×3. 1001 by 13×3.

4. Divide 2500 by 500. 360 by 10×9.

149. Rule for Cancellation.—*Cancel in the dividend and divisor factors common to both, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

PROBLEMS.

150. Divide—

1. $48 \times 3 \times 5$ by $8 \times 7 \times 5$.
2. $50 \times 5 \times 3$ by 15×10.
3. $75 \times 11 \times 2$ by $11 \times 5 \times 2$.
4. 31×25 by $7 \times 5 \times 5$.
5. $4 \times 13 \times 3$ by 39×2.
6. $81 \times 8 \times 7$ by 27×28.

7. How many tons of coal at 9 dollars a ton can be exchanged for 81 barrels of apples at 3 dollars a barrel?

8. How many tons of hay at 24 dollars a ton can be exchanged for 8 thousand feet of boards at 15 dollars a thousand?

9. Divide 11461 by 1400, using the factors 100 and 14.

$14|00)114|61(8\frac{261}{1400}$

112

261

SOLUTION.—Strike off two orders of figures from the right of the dividend and divisor, which divides each by 100, and gives for a new dividend 114 hundreds, and 61 remaining, and for a new divisor, 14 hundreds.

The new dividend 114, divided by the new divisor 14, gives a quotient 8 and a remainder 2, which is 2 hundreds. Annexing to this remainder the first remainder, 61, gives 261, the entire remainder, and the entire quotient is $8\frac{261}{1400}$.

10. Divide 450 by 70, using the factors 10 and 7.

11. Divide 11400 by 600, using the factors 100 and 6.

12. If you should earn 1350 dollars in 300 days, how much would you earn each day?

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151. TEST QUESTIONS.—1. What are the factors of a number? What is a prime number? A composite number? What are the prime factors of a number? What is factoring? A principle of factoring?

2. What is a divisor, or measure, of a number? A common divisor of two or more numbers? The greatest common divisor of two or more numbers? A principle of the greatest common factor or divisor?

3. When is a number a multiple of another? A common multiple of two or more numbers? The least common multiple of two or more numbers? What is a principle of the least common multiple of two or more numbers?

4. What is cancellation? What is a principle in cancellation?

SECTION XIII.

NOTATION OF FRACTIONS.

152.—1. When a cake is divided into two equal parts, what is each of the parts called?

2. Into how many halves can a cake be cut?

3. When an apple is divided into three equal parts, what is each of the parts? What are two of the parts? How many thirds in an apple?

4. When an orange is cut into four equal parts, what is each of the parts? What are two of the parts? Three of the parts? How many fourths in an orange?

5. If a cake be divided into five equal parts, what is each of the parts? What are two of the parts? Three of the parts? Four of the parts?

6. Into how many halves can any thing be divided? Into how many thirds? Fourths? Fifths? Sixths?

7. What is meant by one half of any thing? By one third of any thing? By two thirds? By one fourth? By two fourths? By three fourths?

8. Which are the smaller parts of any thing—halves or thirds? Halves or fourths? Thirds or fourths? Halves or sixths? Thirds or sixths? Fourths or sixths?

9. Halves, thirds, fourths, etc., are expressed by figures, as follows—

One half is written	$\frac{1}{2}$.	Two thirds are written	$\frac{2}{3}$.
One third "	$\frac{1}{3}$.	Three fourths "	$\frac{3}{4}$.
One fourth "	$\frac{1}{4}$.	Four fifths "	$\frac{4}{5}$.
One fifth "	$\frac{1}{5}$.	Six sevenths "	$\frac{6}{7}$.
One twelfth "	$\frac{1}{12}$.	Nine eighths "	$\frac{9}{8}$.
One fifteenth "	$\frac{1}{15}$.	Eleven sixteenths "	$\frac{11}{16}$.
One twentieth "	$\frac{1}{20}$.	Three twenty-fourths "	$\frac{3}{24}$.

10. How many halves of 1 does $\frac{1}{2}$ express? How many thirds of 1 does $\frac{2}{3}$ express? How many fourths of 1 does $\frac{3}{4}$ express?

11. What does the figure 4 under the dividing line in the expression $\frac{3}{4}$ denote?

The number of equal parts into which the unit 1 is divided.

12. What does the figure 3 above the dividing line in the expression $\frac{3}{4}$ denote?

The number of equal parts of the unit 1 which are taken.

13. How is the expression $3\frac{1}{5}$ read?

14. In the expression $3\frac{1}{5}$, what expresses the integer? What the fraction?

15. In the expression $7\frac{5}{7}$, what expresses the integer? What the fraction?

DEFINITIONS.

153. A **Fraction** is a number which represents one or more of the equal parts into which a unit is divided.

154. The **Denominator** of a fraction is the number which shows into how many equal parts the unit is divided.

155. The **Numerator** of a fraction is the number which shows how many equal parts are taken.

156. The **Terms** of a fraction are its numerator and denominator.

157. A **Mixed Number** is a number expressed by an integer and a fraction.

WRITTEN EXERCISES.

158. Write and read—

1. $\frac{6}{7}$.	7. $\frac{10}{19}$.	13. $\frac{17}{3}$.	19. $4\frac{2}{3}$.
2. $\frac{8}{11}$.	8. $\frac{11}{30}$.	14. $\frac{19}{7}$.	20. $7\frac{5}{7}$.
3. $\frac{13}{15}$.	9. $\frac{15}{16}$.	15. $\frac{14}{2}$.	21. $21\frac{1}{4}$.
4. $\frac{2}{19}$.	10. $\frac{1}{19}$.	16. $\frac{101}{13}$.	22. $71\frac{10}{13}$.
5. $\frac{16}{21}$.	11. $\frac{23}{33}$.	17. $\frac{30}{11}$.	23. $99\frac{9}{10}$.
6. $\frac{12}{17}$.	12. $\frac{3}{17}$.	18. $\frac{125}{4}$.	24. $106\frac{3}{17}$.

159. Write in figures—

1. Three fifths.	9. Two twenty-firsts.
2. Four sevenths.	10. Five thirty-seconds.
3. Five eighths.	11. Nineteen eighths.
4. Six tenths.	12. Thirty-one ninths.
5. Nine fourths.	13. Six and six elevenths.
6. Five sixths.	14. Twelve and four fifths.
7. Four ninths.	15. Twenty-one and one fourth.
8. Two elevenths.	16. Nineteen and one thirteenth.

VALUE OF A FRACTION.

160.—1. If you should cut 1 apple into halves, how many halves would there be?

2. If you should cut 5 apples into halves, how many halves would there be? One half of 5 apples is how many halves of 1 apple?

3. How many apples are one half of 5 apples? 5 halves are what part of 5 apples?

4. Is $\frac{1}{2}$ greater or less than 1? Is $\frac{5}{2}$ greater or less than 1? What is the value of $\frac{2}{2}$ in ones? Of $\frac{10}{2}$?

5. Is $\frac{4}{3}$ greater or less than 1? What is the value of $\frac{8}{4}$ in ones? Of $\frac{9}{3}$?

6. Considered as an expression of division, what is the value of $\frac{4}{2}$? Of $\frac{15}{3}$?

7. Considered as an expression of division, what is the value of $\frac{12}{3}$? Of $\frac{14}{7}$?

DEFINITIONS.

161. A **Proper Fraction** is a fraction whose numerator is less than its denominator.

162. An **Improper Fraction** is a fraction whose numerator is not less than its denominator.

163. **Reduction of Fractions** is the process of changing their form or denomination without changing their value.

164. Principle.—*The value of a fraction is the quotient arising from the division of the numerator by the denominator.*

WRITTEN EXERCISES.

165. Write and name the following fractions—

1. $\frac{3}{4}$.	4. $\frac{7}{9}$.	7. $\frac{11}{13}$.	10. $\frac{1}{10}$.
2. $\frac{5}{3}$.	5. $\frac{8}{11}$.	8. $\frac{15}{7}$.	11. $\frac{17}{8}$.
3. $\frac{6}{7}$.	6. $\frac{9}{5}$.	9. $\frac{9}{17}$.	12. $\frac{13}{2}$.

What is the value of—

13. $\frac{12}{2}$?	16. $\frac{10}{5}$?	19. $\frac{17}{4}$?	22. $\frac{16}{7}$?
14. $\frac{15}{3}$?	17. $\frac{8}{2}$?	20. $\frac{21}{5}$?	23. $\frac{25}{6}$?
15. $\frac{20}{4}$?	18. $\frac{35}{7}$?	21. $\frac{19}{8}$?	24. $\frac{47}{8}$?

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166. TEST QUESTIONS.—1. What is a fraction? What is the denominator of a fraction? The numerator of a fraction? What are the terms of a fraction? Take some fraction and show what are its terms.

2. What is a mixed number? Take some mixed number and show what expresses the integer and what the fraction.

3. What is a proper fraction? An improper fraction? What is reduction of fractions? What is the value of a fraction?

SECTION XIV.

REDUCTION OF FRACTIONS.

CASE I.

Integers or Mixed Numbers Reduced to Fractions.

167.—1. If 1 apple be cut into halves, how many halves will there be?

2. In 2 apples, how many halves of 1 apple?

SOLUTION.—Since in 1 apple there are 2 halves, in 2 apples there must be 2 times 2 halves, which are 4 halves.

3. How many halves are there in 3? In 5?

4. In 1 orange, how many thirds of 1 orange? How many fourths of 1 orange?

5. In 1 dollar, how many fifths of 1 dollar? In 6 dollars? In 11 dollars?

6. In $3\frac{1}{2}$ apples, how many halves of 1 apple?

SOLUTION.—Since in 1 apple there are 2 halves, in 3 apples there must be 3 times 2 halves, which are 6 halves; 6 halves and 1 half are 7 halves. Hence, in $3\frac{1}{2}$ apples there are $\frac{7}{2}$ of 1 apple.

7. If you had $5\frac{1}{2}$ dollars to give some poor boys, to how many could you give $\frac{1}{2}$ of 1 dollar each?

8. In $9\frac{1}{2}$ dollars, how many halves? In $6\frac{2}{3}$, how many thirds? In $7\frac{3}{4}$, how many fourths?

9. In $4\frac{1}{5}$, how many fifths? In $10\frac{6}{8}$, how many eighths? In $3\frac{5}{11}$, how many elevenths?.

10. How may a number of ones be changed to halves? To ninths? To twelfths?

WRITTEN EXERCISES.

168.—1. Change 9 to sixths.

6 sixths
9
54 sixths $= \frac{54}{6}$

SOLUTION.—Since in 1 there are 6 sixths, in 9 there must be 9 times 6 sixths, which are 54 sixths $= \frac{54}{6}$.

2. Change 19 to fourths. 25 to tenths.
3. Reduce 65 to sevenths. 73 to eighths.
4. Change $8\frac{5}{9}$ to ninths.

$8\frac{5}{9} = 8 + \frac{5}{9}$
$8 = 72$ *ninths*
$\frac{5}{9} = 5$ "
77 ninths $= \frac{77}{9}$

SOLUTION.—$8\frac{5}{9}$ is equivalent to $8 + \frac{5}{9}$. 8 is equal to 72 ninths, and 72 ninths plus 5 ninths are 77 ninths $= \frac{77}{9}$.

5. Reduce $13\frac{4}{5}$ to fifths. $19\frac{8}{11}$ to elevenths.
6. Reduce $27\frac{1}{3}$ to thirds. $57\frac{5}{13}$ to thirteenths.

169. Rule for Reduction of Integers or Mixed Numbers to Improper Fractions.—*Multiply the integer by the denominator, and, if there be a fractional part, add its numerator to the product. This result, written over the denominator, will be the required fraction.*

PROBLEMS.

170. Reduce to improper fractions—

1. $4\frac{1}{3}$.	4. $14\frac{2}{7}$.	7. $31\frac{1}{7}$.	10. $61\frac{1}{2}$.
2. $7\frac{4}{9}$.	5. $16\frac{3}{13}$.	8. $20\frac{1}{10}$.	11. $13\frac{14}{15}$.
3. $11\frac{5}{11}$.	6. $19\frac{4}{15}$.	9. $31\frac{2}{11}$.	12. $16\frac{2}{19}$.

CASE II.

Fractions Reduced to Integers or Mixed Numbers.

171.—1. In 5 halves of an apple are how many apples?

SOLUTION.—Since 2 halves of an apple equal 1 apple, 5 halves of an apple must equal as many apples as 2 halves are contained times in 5 halves, which are $2\frac{1}{2}$ times.

Hence, in 5 halves of an apple there are $2\frac{1}{2}$ apples.

2. How many oranges will be required to give 7 boys half an orange each?

3. If John can gather 1 third of a bushel of berries in 1 day, how many thirds of a bushel can he gather in 6 days? How many bushels are 6 thirds of a bushel?

4. If a man can spend 1 fourth of a dollar in 1 day, how many dollars can he spend in 8 days?

5. How many ones in $\frac{10}{5}$? In $\frac{35}{5}$? In $\frac{15}{5}$? In $\frac{32}{7}$?

6. How many ones in $\frac{17}{3}$? In $\frac{41}{9}$? In $\frac{73}{12}$? In $\frac{81}{9}$?

WRITTEN EXERCISES.

172.—1. Change $\frac{112}{4}$ to an equivalent integer.

$\frac{112}{4} = 28$

SOLUTION.—Since 4 fourths equal one, 112 fourths must equal as many ones as 4 is contained times in 112, which are 28 times. Hence, $\frac{112}{4} = 28$.

2. Change $\frac{455}{5}$ to an equivalent integer.

3. Change $\frac{584}{8}$ to an equivalent integer.

4. Change $\frac{573}{5}$ to an equivalent mixed number.

$\frac{573}{5} = 114\frac{3}{5}$

SOLUTION.—Since 5 fifths equal one, 573 fifths must equal as many ones as 5 is contained times in 573, which is $114\frac{3}{5}$ times. Hence, $\frac{573}{5} = 114\frac{3}{5}$.

5. Change $\frac{148}{11}$ to a mixed number.

6. Change $\frac{2223}{13}$ to a mixed number.

173. Rule for Reduction of an Improper Fraction to an Integer or Mixed Number.—*Divide the numerator of the fraction by the denominator.*

PROBLEMS.

174. Change to integers or mixed numbers—

1. $\frac{154}{11}$.	4. $\frac{401}{5}$.	7. $\frac{553}{7}$.	10. $\frac{189}{9}$.
2. $\frac{133}{9}$.	5. $\frac{147}{9}$.	8. $\frac{589}{19}$.	11. $\frac{913}{3}$.
3. $\frac{119}{7}$.	6. $\frac{128}{5}$.	9. $\frac{517}{21}$.	12. $\frac{2669}{37}$.

13. What is the value of $\frac{2048}{8}$? Of $\frac{315}{4}$ of a yard?

CASE III.

Fractions Reduced to Higher or Lower Terms.

175.—1. If an orange be divided into two equal parts, and each of these parts be divided into two equal parts, what part of the orange is one of the pieces?

2. One half is equal to how many fourths?

SOLUTION.—Since one equals 4 fourths, 1 half must equal one half of 4 fourths, or 2 fourths.

3. If you divide an orange into two equal parts, and each of these parts be divided into three equal parts, what part of the orange will one of the pieces be?

4. One half is equal to how many sixths? Eighths?

5. In what equivalent fractions can halves be expressed?

In fourths, sixths, eighths, tenths, etc.

6. The denominators of these fractions are multiples of what number?

Of 2, the denominator of $\frac{1}{2}$.

7. If an apple be divided into three equal parts, and each of these parts into two equal parts, what part of the apple will one of these pieces be?

8. One third is equal to how many sixths? Ninths?

9. In what equivalent fractions can thirds be expressed?

10. The denominators of thirds, sixths, twelfths, etc., are multiples of what number?

11. The sixths of a number are how many times the thirds? The twelfths are how many times the thirds?

12. If you multiply both terms of $\frac{1}{3}$ by 2, what have you? If you multiply both terms of $\frac{1}{3}$ by 4, what have you?

DEFINITION.

176. A fraction is reduced to **Higher Terms** when its numerator and denominator are changed to higher numbers without changing its value.

177. Principle.—*Multiplying both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

178.—1. Reduce $\frac{5}{6}$ to twenty-fourths.

$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

SOLUTION.—Since 24, the required denominator, is 4 times as large as 6, the given denominator, we multiply both terms of the given fraction by 4, which gives $\frac{20}{24}$, the fraction required.

2. Reduce $\frac{7}{8}$ to forty-eighths. $\frac{3}{16}$ to sixty-fourths.

3. Reduce $\frac{5}{11}$ to fifty-fifths. $\frac{7}{15}$ to seventy-fifths.

179. Rule for Reduction of Fractions to Higher Terms.—*Multiply both terms of the fraction by such a number as will give the required denominator.*

PROBLEMS.

180. Reduce—

1. $\frac{7}{8}$ to seventy-seconds.
2. $\frac{1}{6}$ to sixtieths.
3. $\frac{5}{9}$ to eighty-firsts.
4. $\frac{4}{13}$ to sixty-fifths.
5. $\frac{1}{19}$ to fifty-sevenths.
6. $\frac{9}{27}$ to one-hundred-eighths.

CASE IV.

Fractions Reduced to Lower Terms.

181.—1. How many fourths are equal to $\frac{6}{8}$?

SOLUTION.—Since 2 eighths equal 1 fourth, 6 eighths must equal as many fourths as 2 eighths are contained times in 6 eighths, which are 3 times. Hence $\frac{6}{8} = \frac{3}{4}$.

2. How many fifths are equal to $\frac{3}{15}$? To $\frac{4}{20}$?

3. How many thirds are equal to $\frac{6}{9}$? To $\frac{18}{27}$?

4. Express $\frac{6}{24}$ in parts two times as great.

5. Express $\frac{9}{45}$ by a fraction having a denominator one ninth as large.

6. Reduce $\frac{9}{27}$ to thirds. $\frac{12}{21}$ to sevenths. $\frac{48}{64}$ to eighths.

7. Reduce $\frac{24}{32}$ to fourths. $\frac{27}{72}$ to ninths. $\frac{50}{80}$ to tenths.

8. $\frac{10}{18}$ is equal to $\frac{5}{9}$. By what number must you divide both the numerator and denominator of $\frac{10}{18}$ to express the same value by $\frac{5}{9}$?

9. To what fraction having smaller or lower terms may $\frac{6}{8}$ be changed? May $\frac{18}{24}$?

10. What common factors greater than 1 have the terms of $\frac{6}{8}$? Have the terms of $\frac{18}{24}$?

11. When all the factors greater than 1 common to the numerator and denominator of $\frac{18}{24}$ are cancelled, what will be the fraction?

DEFINITION.

182. A fraction is in its **Lowest Terms** when its numerator and denominator have no common factor greater than 1.

183. Principle.—*Dividing both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

184.—1. Reduce $\frac{20}{24}$ to its lowest terms.

$$\frac{20}{24} = \frac{5 \times \cancel{2} \times \cancel{2}}{3 \times \cancel{2} \times \cancel{2} \times 2} = \frac{5}{3 \times 2} = \frac{5}{6}$$

SOLUTION.—The factors of the numerator 20 are 5, 2 and 2; the factors of the denominator 24 are 3, 2, 2 and 2. Since the fraction in its lowest terms can have no factor greater than 1 common to both its terms, cancel the factors 2 and 2, leaving $\frac{5}{3 \times 2} = \frac{5}{6}$, which is the fraction in its lowest terms.

2. Reduce $\frac{42}{48}$ to its lowest terms. $\frac{12}{64}$ to its lowest terms.

3. Reduce $\frac{25}{55}$ to its lowest terms. $\frac{35}{75}$ to its lowest terms.

185. Rule for Reduction of Fractions to their Lowest Terms.—*Cancel in both terms of the given fraction all common factors.*

PROBLEMS.

186. Reduce to their lowest terms—

1. $\frac{63}{72}$.
2. $\frac{10}{60}$.
3. $\frac{45}{81}$.
4. $\frac{20}{65}$.
5. $\frac{3}{57}$.
6. $\frac{36}{108}$.
7. $\frac{6}{54}$.
8. $\frac{21}{105}$.

CASE V.

Fractions Reduced to a Common Denominator.

187.—1. William has $\frac{1}{2}$ of a dollar, and John has $\frac{12}{16}$. How many fourths of a dollar has each?

SOLUTION.—1 half of a dollar is equal to 2 fourths of a dollar, and 12 sixteenths of a dollar is equal to 3 fourths of a dollar. Hence, William has $\frac{2}{4}$ of a dollar, and John $\frac{3}{4}$ of a dollar.

2. A father gave to one of his sons $\frac{1}{3}$ of a pine-apple, and to another $\frac{2}{5}$. How many fifteenths of a pine-apple did each receive?

3. Change $\frac{5}{6}$ and $\frac{2}{3}$ to eighteenths. $\frac{3}{7}$ and $\frac{1}{2}$ to fourteenths.

4. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to fractions having the same denominator.

5. Reduce $\frac{1}{4}$ and $\frac{5}{7}$ to fractions having the same denominator.

6. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having a common denominator. To fractions having the least common denominator.

7. What is a common multiple of 4 and 6, the denominators of $\frac{3}{4}$ and $\frac{5}{6}$? What is their least common multiple?

8. Reduce $\frac{2}{3}$ and $\frac{5}{9}$ to ninths. What is the least common multiple of the denominators of $\frac{2}{3}$ and $\frac{5}{9}$?

9. What is the least common denominator of $\frac{2}{3}$ and $\frac{5}{9}$?

10. What is the least common denominator of $\frac{1}{2}$, $\frac{1}{5}$ and $\frac{3}{10}$?

188. Principle.—*The least common denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

189.—1. Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to fractions having a common denominator.

$\frac{3\times2}{4\times2}=\frac{6}{8}$

SOLUTION.—Since 2 times the number of fourths must be the number of eighths, multiply both terms of $\frac{3}{4}$ by 2, which gives, as its equivalent, $\frac{6}{8}$. Hence, $\frac{6}{8}$ and $\frac{5}{8}$ are the fractions required.

2. Reduce $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{5}{12}$ to fractions having the least common denominator.

$\frac{1\times12}{2\times12}=\frac{12}{24}$
$\frac{3\times3}{8\times3}=\frac{9}{24}$
$\frac{5\times2}{12\times2}=\frac{10}{24}$

SOLUTION.—The least common multiple of the denominators of the fractions is 24. Hence, 24 must be their least common denominator.

Since the required denominator 24 is 12 times the denominator of $\frac{1}{2}$, multiply both its terms by 12, reducing it to $\frac{12}{24}$. Since the required denominator is 3 times the denominator of $\frac{3}{8}$, multiply both its terms by 3, reducing it to $\frac{9}{24}$; and since the required denominator is 2 times the denominator of $\frac{5}{12}$, multiply both its terms by 2, reducing it to $\frac{10}{24}$. Hence, $\frac{12}{24}$, $\frac{9}{24}$ and $\frac{10}{24}$ are the fractions required.

3. Reduce $\frac{5}{6}$ and $\frac{9}{11}$ to fractions having a common denominator.

4. Reduce $\frac{1}{3}$, $\frac{2}{9}$ and $\frac{5}{6}$ to fractions having the least common denominator.

190. **Rule for Reducing Fractions to a Common Denominator.**—*Multiply both terms of each fraction by any number that will make their denominators alike.*

191. **Rule for Reducing Fractions to the Least Common Denominator.**—*Find the least common multiple of all the denominators for the least common denominator, and multiply both terms of each fraction by such a number as will reduce it to that denominator.*

PROBLEMS.

192.—1. Change $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having a common denominator.

2. Change $\frac{3}{5}$, $\frac{1}{10}$ and $\frac{4}{15}$ to fractions having a common denominator.

3. Change $\frac{6}{11}$, $\frac{5}{22}$ and $\frac{7}{44}$ each to eighty-eighths.

Reduce to the least common denominator—

4. $\frac{2}{3}$, $\frac{4}{9}$ and $\frac{7}{12}$.
5. $\frac{1}{2}$, $\frac{5}{6}$ and $\frac{3}{8}$.
6. $\frac{2}{5}$, $\frac{7}{8}$ and $\frac{9}{10}$.
7. $\frac{2}{7}$, $\frac{3}{10}$ and $\frac{11}{14}$.
8. $\frac{1}{6}$, $\frac{9}{14}$ and $\frac{8}{21}$.
9. $\frac{4}{5}$, $\frac{7}{8}$ and $\frac{15}{20}$.
10. $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.
11. $\frac{2}{15}$, $\frac{11}{18}$ and $\frac{17}{20}$.
12. $\frac{11}{16}$, $\frac{5}{20}$ and $\frac{21}{45}$.

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193. TEST QUESTIONS.—1. When is a fraction reduced to higher terms? When to lower terms?

2. How do you reduce an integer or a mixed number to an improper fraction? How do you reduce an improper fraction to an integer or a mixed number?

3. What effect upon the value of a fraction has the multiplying of both terms by the same number? The dividing of both terms by the same number?

4. How do you reduce fractions to higher terms? To lower terms?

5. When have fractions a common denominator? What is the least common denominator of two or more fractions? How do you reduce fractions to fractions having a common denominator? The least common denominator?

SECTION XV.

ADDITION OF FRACTIONS.

194.—1. John has $\frac{2}{8}$ of a dollar, and his brother has $\frac{3}{8}$. How many eighths have both?

2. If Jane has $\frac{3}{5}$ of an orange, and Susan has $\frac{4}{5}$, how many fifths have both?

3. How many ones are $\frac{3}{5}$ and $\frac{4}{5}$? $\frac{11}{12}$ and $\frac{7}{12}$?

4. How do you add fractions having a common denominator?

By adding their numerators and writing their sum over the denominator.

5. Jason has $\frac{3}{4}$ of an acre planted with corn, and $\frac{2}{3}$ of an acre with potatoes. How many acres has he planted?

SOLUTION.—He has planted $\frac{3}{4}$ of an acre $+$ $\frac{2}{3}$ of an acre. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{2}{3}$ to $\frac{8}{12}$; and $\frac{9}{12}$ and $\frac{8}{12}$ are $\frac{17}{12}$, or $1\frac{5}{12}$. Hence, he has planted $1\frac{5}{12}$ acres.

6. How can fractions which have different denominators be added?

By first reducing them to a common denominator.

7. Mary has $\frac{5}{8}$ of an apple and Ella has $\frac{3}{4}$ of an apple. How many apples have both?

8. Henry gave $\frac{7}{10}$ of a dollar for a book and $\frac{7}{8}$ of a dollar for a knife. How many dollars did he give for both?

9. How many are $\frac{5}{6}$ and $\frac{7}{8}$? $\frac{4}{5}$ and $\frac{3}{7}$?

10. A man having undertaken a piece of work, performed $\frac{1}{2}$ of it the first day, $\frac{1}{3}$ of it the second, and $\frac{1}{4}$ of it the third. How much of it did he perform in the three days?

11. How many are $\frac{5}{6}$ and $\frac{7}{8}$? $\frac{4}{5}$ and $\frac{3}{7}$?

12. A man bought $1\frac{1}{2}$ bushels of corn at one time and $3\frac{3}{8}$ bushels at another. How many bushels did he buy in all?

195. Principle.—*Fractions must express like parts, or have a common denominator, before they can be added.*

WRITTEN EXERCISES.

196.—1. What is the sum of $\frac{5}{7}$, $\frac{3}{7}$ and $\frac{6}{7}$?

$\frac{5}{7}+\frac{3}{7}+\frac{6}{7}=\frac{14}{7}=2$

SOLUTION.—Since the fractions all express like parts, their sum may be found by adding their numerators. 5 sevenths + 3 sevenths + 6 sevenths = $\frac{14}{7}$, or 2.

2. What is the sum of $\frac{15}{20}$, $\frac{13}{20}$ and $\frac{19}{20}$?
3. What is the sum of $\frac{31}{32}$, $\frac{29}{32}$ and $\frac{11}{32}$?
4. What is the sum of $\frac{1}{2}$, $\frac{4}{5}$ and $\frac{3}{8}$?

$\frac{1}{2}=\frac{20}{40}$; $\frac{4}{5}=\frac{32}{40}$; $\frac{3}{8}=\frac{15}{40}$

$\frac{20}{40}+\frac{32}{40}+\frac{15}{40}=\frac{67}{40}=1\frac{27}{40}$

SOLUTION. — Reducing the fractions to their least common denominator, we have $\frac{20}{40}+\frac{32}{40}+\frac{15}{40}=\frac{67}{40}$, or $1\frac{27}{40}$, which is the sum required.

5. What is the sum of $\frac{5}{9}$, $\frac{7}{8}$ and $\frac{11}{14}$?
6. What is the sum of $\frac{3}{5}$, $\frac{9}{10}$ and $\frac{14}{15}$?
7. What is the sum of $3\frac{3}{4}$, $5\frac{1}{2}$ and 11?

$3\frac{3}{4}=3\frac{6}{8}$

$5\frac{1}{2}=5\frac{4}{8}$

$11=11$

Sum, $20\frac{2}{8}=20\frac{1}{4}$

SOLUTION.—Reducing the fractions to their least common denominator and adding their numerators, we have as their sum $\frac{10}{8}=1\frac{2}{8}$. Write the $\frac{2}{8}$ under the fractions, and add the 1 with the integers, giving $20\frac{2}{8}=20\frac{1}{4}$, the sum required.

8. What is the sum of $11\frac{3}{4}$, $15\frac{7}{8}$ and $5\frac{1}{2}$?

197. **Rule for Addition of Fractions.**—*Reduce the fractions to equivalent fractions having a common denominator; add their numerators, and write under the sum the common denominator.*

If there be mixed numbers or integers, add the fractions and integers separately, and unite the results.

PROBLEMS.

198. What is the sum of—

1. $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{11}{12}$?
2. $\frac{4}{7}$, $\frac{3}{14}$ and $\frac{1}{16}$?
3. $\frac{9}{25}$, $\frac{16}{25}$ and $\frac{11}{25}$?
4. $\frac{5}{20}$, $\frac{7}{15}$ and $\frac{3}{10}$?
5. $\frac{3}{8}$, $\frac{5}{9}$ and 5?
6. 8, $\frac{3}{11}$ and $2\frac{1}{22}$?
7. $3\frac{1}{2}$, $4\frac{1}{5}$ and 33?
8. $\frac{11}{12}$, $\frac{5}{24}$ and $1\frac{5}{12}$?
9. What is the sum of $\frac{7}{9} + \frac{5}{18} + \frac{2}{3} + 1\frac{1}{2}$?
10. What is the sum of $3\frac{1}{4} + 2\frac{1}{2} + \frac{3}{4} + \frac{5}{6}$?
11. A farmer has in one bin $31\frac{3}{4}$ bushels of wheat, in a second bin, $15\frac{3}{8}$ bushels, and in a third, $16\frac{1}{2}$ bushels. How many bushels has he in the three bins?
12. If you should spend $6\frac{4}{15}$ hours in study, $3\frac{1}{20}$ hours in play, and $11\frac{1}{4}$ hours in sleep, how many hours would you spend in all?

SECTION XVI.

SUBTRACTION OF FRACTIONS.

199.—1. John and his brother have $\frac{5}{8}$ of an apple. If John has $\frac{3}{8}$, what part of the apple has his brother?

2. Jane has $\frac{7}{5}$ of an orange; if she should give Susan $\frac{4}{5}$, what part of the orange would she have left?

3. How many fifths are $\frac{7}{5}$ less $\frac{4}{5}$? $\frac{1}{5}$ less $\frac{7}{5}$?

4. How, then, do you subtract one fraction from another when both have a common denominator?

5. If $\frac{3}{5}$ of an apple be taken from it, what part will remain?

6. How much is 1 less $\frac{1}{3}$? 2 less $\frac{1}{3}$? 3 less $\frac{2}{3}$?

7. If a cord of wood cost $\frac{5}{8}$ of a dollar less than 6 dollars, what is its cost?

8. How much is $\frac{1}{2}$ less $\frac{2}{7}$?

SOLUTION.—$\frac{1}{2}$ equals $\frac{7}{14}$, and $\frac{2}{7}$ equals $\frac{4}{14}$; and $\frac{7}{14}$ less $\frac{4}{14}$ is $\frac{3}{14}$.

9. A man worked $\frac{9}{10}$ of a day, and his son $\frac{4}{5}$ of a day. How much longer did the man work than his son?

10. How much is $\frac{1}{4}$ less $\frac{1}{7}$? $\frac{1}{3}$ less $\frac{2}{9}$? $\frac{1}{2}$ less $\frac{2}{5}$?

11. How do you prepare fractions having different denominators for subtraction?

12. If a man owning $\frac{15}{16}$ of a mill should sell $\frac{3}{4}$ of the mill, what part of it would he have left?

13. How many are $2\frac{1}{9}$ less $\frac{2}{3}$? $5\frac{3}{4}$ less $2\frac{1}{2}$?

14. What number must be added to $3\frac{3}{8}$ to make 9?

200. Principle.—*Fractions must express like parts, or have a common denominator, before they can be subtracted.*

WRITTEN EXERCISES.

201.—1. What is the difference between $\frac{2}{3}$ and $\frac{2}{9}$?

$\frac{2}{3} - \frac{2}{9} = \frac{6}{9} - \frac{2}{9} = \frac{6-2}{9} = \frac{4}{9}$

SOLUTION. — Reducing the fractions to their least common denominator and finding the difference between their numerators, we have $\frac{4}{9}$, the difference required.

2. Find the difference between $\frac{5}{28}$ and $\frac{3}{7}$.

3. Find the difference between $\frac{4}{15}$ and $\frac{7}{8}$.

4. Find the difference between $8\frac{1}{4}$ and $5\frac{2}{3}$.

$8\frac{1}{4} = 8\frac{3}{12} = 7\frac{15}{12}$

$5\frac{2}{3} = 5\frac{8}{12} = 5\frac{8}{12}$

Difference, $2\frac{7}{12}$

SOLUTION.—Reduce the fractions to fractions having the least common denominator. Since $\frac{8}{12}$ cannot be taken from $\frac{3}{12}$, we take 1 one, or $\frac{12}{12}$, from the 8 ones, leaving 7 ones, and adding the $\frac{12}{12}$ to the $\frac{3}{12}$, have $\frac{15}{12}$; $5\frac{8}{12}$ from $7\frac{15}{12}$ leaves $2\frac{7}{12}$, the difference required.

5. Find the difference between $14\frac{1}{5}$ and $6\frac{3}{4}$.

6. Find the difference between $28\frac{3}{7}$ and $9\frac{1}{14}$.

202. Rule for Subtraction of Fractions.—*Reduce the fractions to equivalent fractions having a common denominator, and write the difference of the numerators over the common denominator.*

If there be mixed numbers, subtract first the fractional part of the subtrahend, and then the integral part, and unite the results.

PROBLEMS.

203. Subtract—

1. $\frac{1}{2}\frac{3}{4}$ from $\frac{3}{4}$.
2. $\frac{7}{18}$ from $\frac{5}{9}$.
3. $\frac{11}{31}$ from $\frac{1}{2}$.
4. $\frac{3}{14}$ from $\frac{11}{21}$.
5. $\frac{5}{12}$ from $\frac{5}{11}$.
6. $\frac{21}{34}$ from $\frac{15}{17}$.
7. $13\frac{1}{9}$ from $16\frac{1}{4}$.
8. $11\frac{1}{3}$ from $19\frac{1}{2}$.
9. $33\frac{15}{16}$ from 100.
10. If a farmer should sell $24\frac{7}{16}$ acres from a lot containing $66\frac{3}{4}$ acres, how many acres would be left?
11. What is the difference between $85\frac{1}{2}$ and $83\frac{1}{5}$?

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204. TEST QUESTIONS.—1. What is the principle in addition of fractions? How do you add when all the numbers are fractions? When there are fractions and mixed numbers or integers?

2. What is the principle in subtraction of fractions? How do you subtract when the minuend and subtrahend are fractions? When the minuend and subtrahend are mixed numbers?

SECTION XVII.

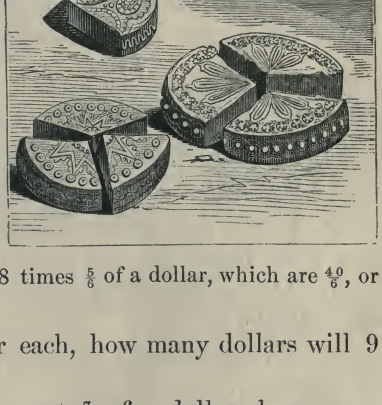
MULTIPLICATION OF FRACTIONS.

CASE I.

Fractions Multiplied by Integers.

205.—1. If you should give $\frac{1}{6}$ of a melon to each of 5 boys, how many sixths of a melon would they all have?

2. If you should give $\frac{1}{3}$ of a cake to each of 7 boys, how many thirds of a cake would be required? How many cakes?



3. 7 times $\frac{1}{3}$ are how many times 1?

4. If 1 yard of cloth cost $\frac{5}{6}$ of a dollar, how many dollars will 8 yards cost?

SOLUTION. — Since 1 yard of cloth cost $\frac{5}{6}$ of a dollar, 8 yards must cost 8 times $\frac{5}{6}$ of a dollar, which are $\frac{40}{6}$, or $\frac{20}{3}$, equal to $6\frac{2}{3}$ dollars.

5. At $\frac{7}{10}$ of a dollar each, how many dollars will 9 hats cost?

6. If 1 pound of tea cost $\frac{7}{8}$ of a dollar, how many dollars will 7 pounds cost?

7. 7 times $\frac{7}{8}$ are how many times 1?

8. If a horse eat $\frac{11}{12}$ of a bushel of grain in 1 week, how many bushels will he eat in 10 weeks?

9. How many are 6 times $\frac{3}{5}$? 7 times $\frac{5}{9}$?

10. How many are 8 times $\frac{4}{11}$? 5 times $\frac{10}{11}$?

11. If $\frac{7}{9}$ of an acre will pasture a cow, how many acres will pasture 12 cows?

12. How many are 11 times $\frac{7}{8}$? 12 times $\frac{4}{7}$?

13. If $\frac{7}{8}$ of a yard of cloth is $\frac{1}{7}$ of what is required for a suit of clothes, how many yards will be required for the suit?

14. If you can earn $\frac{2}{3}\frac{2}{3}$ of a dollar in 1 day, how many dollars can you earn in 11 days?

15. Multiplying the numerator of $\frac{2}{3}\frac{2}{3}$ by 11 gives $\frac{2}{3}\frac{2}{3}$, or $\frac{2}{3}$; dividing the denominator of $\frac{2}{3}\frac{2}{3}$ by 11 gives $\frac{2}{3}$; in each case the fraction is multiplied by 11. How, then, may a fraction be multiplied by an integer?

16. Multiply $\frac{5}{2}\frac{5}{4}$ by 8. $\frac{3}{2}\frac{3}{8}$ by 7. $\frac{7}{1}\frac{7}{0}$ by 5.

17. How many are 5 times $6\frac{1}{2}$?

SOLUTION.—5 times 6 are 30, and 5 times $\frac{1}{2}$ are $\frac{5}{2}$, or $2\frac{1}{2}$, which added to 30 gives $32\frac{1}{2}$, the answer required.

18. If 1 ton of coal cost $5\frac{5}{8}$ dollars, how many dollars will 7 tons cost?

19. If you can gather $6\frac{3}{4}$ quarts of berries in 1 day, how many quarts can you gather in 6 days?

20. At the rate of $3\frac{2}{3}$ miles in 1 hour, how far can you walk in 8 hours?

21. How much must be given for 11 pounds of tea, at $1\frac{3}{8}$ dollars a pound?

22. What will 5 yards of cloth cost, at $3\frac{3}{4}$ dollars a yard?

206. Principle.—*Multiplying the numerator, or dividing the denominator, by any number, multiplies the fraction by that number.*

WRITTEN EXERCISES.

207.—1. Multiply $\frac{11}{16}$ by 8.

$$\frac{11}{16}\times 8=\frac{11\times 8}{16}=\frac{88}{16}=5\frac{8}{16}=5\frac{1}{2}$$

Or,

$$\frac{11}{16}\times 8=\frac{11}{16\div 8}=\frac{11}{2}=5\frac{1}{2}$$

SOLUTION.—8 times 11 sixteenths are $\frac{88}{16}$, which, reduced, is $5\frac{8}{16}$, or $5\frac{1}{2}$, the result required.

Or, since dividing the denominator of a fraction multiplies the fraction, 8 times $\frac{11}{16}=\frac{11}{2}$, or $5\frac{1}{2}$, the same result.

2. Multiply $11\frac{3}{4}$ by 7.

$$11\tfrac{3}{4}=11+\tfrac{3}{4}$$
$$11\times 7=77$$
$$\tfrac{3}{4}\times 7=\tfrac{21}{4}=5\tfrac{1}{4}$$
$$82\tfrac{1}{4}$$

SOLUTION.—Since $11\frac{3}{4}$ equals $11+\frac{3}{4}$, the product of $11\frac{3}{4}$ by 7 is the same as 7 times 11 plus 7 times $\frac{3}{4}$. 7 times 11 are 77, and 7 times $\frac{3}{4}$ are $\frac{21}{4}$, or $5\frac{1}{4}$. $77+5\frac{1}{4}=82\frac{1}{4}$, the product required.

3. Multiply $\frac{19}{81}$ by 9. $\frac{43}{101}$ by 17. $\frac{103}{300}$ by 15.

4. Find the product of $25\frac{7}{8}$ by 16.

208. Rules for Multiplying a Fraction by an Integer.—*Multiply the numerator, or divide the denominator, by the integer.*

If the multiplicand be a mixed number, multiply the integer and fraction separately, and add the products.

PROBLEMS.

209. Multiply—

1. $\frac{7}{10}$ by 9.	5. $\frac{14}{45}$ by 15.	9. $3\frac{5}{9}$ by 17.
2. $\frac{10}{13}$ by 20.	6. $\frac{19}{63}$ by 9.	10. $7\frac{3}{7}$ by 14.
3. $\frac{7}{9}$ by 25.	7. $\frac{5}{16}$ by 8.	11. $4\frac{2}{11}$ by 31.
4. $\frac{3}{26}$ by 13.	8. $\frac{17}{91}$ by 18.	12. $19\frac{2}{7}$ by 63.

CASE II.

Integers Multiplied by Fractions.

210.—1. $\frac{1}{4}$ of 3 inches is what part of 1 inch?

SOLUTION.—$\frac{1}{4}$ of 3 inches must be 3 times $\frac{1}{4}$ of 1 inch, or $\frac{3}{4}$ of 1 inch.

2. $\frac{1}{5}$ of 3 apples is how many apples?

3. $\frac{1}{9}$ of 8 bushels is how many bushels?

4. Arthur has 5 dollars, and James has $\frac{2}{3}$ as many. How many dollars has James?

SOLUTION.—James has $\frac{2}{3}$ of 5 dollars. Since $\frac{1}{3}$ of 5 dollars is $\frac{5}{3}$ of 1 dollar, $\frac{2}{3}$ of 5 dollars must be 2 times $\frac{5}{3}$, which are $\frac{10}{3}$, or $3\frac{1}{3}$ dollars. Therefore James has $3\frac{1}{3}$ dollars.

5. How much will $\frac{3}{8}$ of a yard of cloth cost, at the rate of 7 dollars a yard?

6. $\frac{3}{8}$ of 7 is what number?

7. If a man can do a piece of work in 45 days, in how many days can he do $\frac{2}{3}$ of it?

8. $\frac{2}{3}$ of 45 is what number?

9. If a barrel of flour is worth 16 dollars, how much is $\frac{7}{8}$ of a barrel worth?

10. $\frac{5}{7}$ of 42 is what number?

11. Multiply 9 by $\frac{2}{5}$. 5 by $\frac{3}{7}$. 8 by $\frac{4}{9}$.

12. At 6 dollars a ton, what will $5\frac{3}{4}$ tons of coal cost?

SOLUTION.—Since 1 ton costs 6 dollars, $5\frac{3}{4}$ tons will cost $5\frac{3}{4}$ times 6 dollars; 5 times 6 dollars are 30 dollars, and $\frac{3}{4}$ of 6 dollar are $\frac{18}{4}$, or $4\frac{1}{2}$ dollars; 30 dollars and $4\frac{1}{2}$ dollars are $34\frac{1}{2}$ dollars, the cost required.

13. At 5 dollars a yard, what will $9\frac{5}{8}$ yards of cloth cost?

14. Multiply 8 by $4\frac{3}{8}$. 7 by $3\frac{5}{7}$. 9 by $2\frac{6}{11}$.

15. If 6 men can do a piece of work in $4\frac{3}{4}$ days, how many men will it take to do it in 1 day?

16. Multiply 4 by $6\frac{7}{11}$. 8 by $8\frac{2}{5}$. 5 by $7\frac{7}{12}$.

211. Principle.—*A number is multiplied by a fraction by obtaining such a part of the number as the fraction indicates.*

WRITTEN EXERCISES.

212.—1. Multiply 35 by $\frac{5}{7}$, or find $\frac{5}{7}$ of 35.

$$35 \times \tfrac{5}{7} = \frac{35 \times 5}{7} = 25$$

Or,

$$35 \times \tfrac{5}{7} = \frac{35}{7} \times 5 = 25$$

SOLUTION.—Since $\frac{5}{7} = \frac{1}{7}$ of 5, $\frac{5}{7}$ times 35 must equal $\frac{1}{7}$ of 5 times 35, or $\frac{35 \times 5}{7}$, which, by canceling, or $\frac{5 \times 5}{1}$, or 25.

Or, since $\frac{5}{7} = 5$ times $\frac{1}{7}$, we find $\frac{5}{7}$ of 35 by taking 5 times $\frac{1}{7}$ of 35; $\frac{1}{7}$ of 35 is 5, and 5 times 5 are 25.

2. Multiply 66 by $\frac{4}{11}$, or find $\frac{4}{11}$ of 66.

3. Multiply 75 by $\frac{13}{15}$, or find $\frac{13}{15}$ of 75.

213. Rules for Multiplying an Integer by a Fraction.—*Multiply the integer by the numerator of the fraction, and divide the product by the denominator. Or,*

Divide the integer by the denominator of the fraction, and multiply the quotient by the numerator.

PROBLEMS.

214. Multiply—

1. 72 by $\frac{5}{12}$.
2. 96 by $\frac{3}{6}$.
3. 105 by $\frac{5}{17}$.
4. 112 by $\frac{4}{9}$.
5. 215 by $\frac{2}{17}$.
6. 360 by $\frac{7}{9}$.
7. 327 by $\frac{5}{16}$.
8. 516 by $\frac{3}{4}$.
9. 819 by $\frac{2}{21}$.

CASE III.

Fractions Multiplied by Fractions.

215.—1. If $\frac{1}{3}$ of a pear be separated into two equal parts, what part of the pear will one of those parts be?

2. $\frac{1}{2}$ of $\frac{1}{3}$ is what part of 1?

3. If $\frac{2}{3}$ of an orange be equally shared by two boys, what part of the orange will each receive?

4. $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1?

SOLUTION.—$\frac{1}{2}$ of $\frac{2}{3}$ is equal to 2 times $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, and 2 times $\frac{1}{6}$ are $\frac{2}{6}$, or $\frac{1}{3}$, the part required.

5. What is $\frac{1}{2}$ of $\frac{3}{4}$? $\frac{1}{4}$ of $\frac{5}{6}$? $\frac{1}{5}$ of $\frac{7}{8}$?

6. What is $\frac{1}{2}$ of $\frac{5}{6}$? $\frac{1}{3}$ of $\frac{5}{6}$? $\frac{1}{6}$ of $\frac{4}{5}$?

7. A man owning $\frac{7}{8}$ of a ship sold $\frac{3}{4}$ of his share. What part of the ship did he sell?

SOLUTION.—He sold $\frac{3}{4}$ of $\frac{7}{8}$ of the ship; $\frac{3}{4}$ of $\frac{7}{8}$ is equal to 3 times $\frac{1}{4}$ of $\frac{7}{8}$; $\frac{1}{4}$ of $\frac{7}{8}$ is equal to $\frac{7}{32}$, and 3 times $\frac{7}{32}$ are $\frac{21}{32}$. Therefore, he sold $\frac{21}{32}$ of the ship.

8. When a man had traveled $\frac{9}{10}$ of a mile he had still to travel a distance equal to $\frac{4}{5}$ of that gone over. How far had he still to travel, and how far did he travel in all?

9. What is $\frac{3}{7}$ of $\frac{4}{5}$? $\frac{2}{3}$ of $\frac{7}{9}$? $\frac{5}{6}$ of $\frac{7}{8}$?

10. What is $\frac{3}{4}$ of $\frac{2}{11}$? $\frac{3}{10}$ of $\frac{3}{4}$? $\frac{6}{7}$ of $\frac{3}{4}$?

11. What will $\frac{3}{8}$ of a bushel of corn cost, at $\frac{9}{10}$ of a dollar a bushel?

12. At $2\frac{1}{2}$ dollars a yard, what is the cost of $\frac{5}{6}$ of a yard of cloth?

SOLUTION.—Since 1 yard costs $2\frac{1}{2}$ dollars, $\frac{5}{6}$ of a yard will cost $\frac{5}{6}$ of $2\frac{1}{2}$ dollars. $2\frac{1}{2}$ dollars are equal to $\frac{5}{2}$ of a dollar; $\frac{5}{6}$ of $\frac{5}{2}$ are $\frac{25}{12}$, or $2\frac{1}{12}$. Hence, $\frac{5}{6}$ of a yard will cost $2\frac{1}{12}$ dollars.

13. If a cord of wood cost $3\frac{1}{4}$ dollars, how much will $\frac{3}{4}$ of a cord cost?

14. What is $\frac{2}{5}$ of $5\frac{1}{4}$? $\frac{2}{7}$ of $3\frac{1}{3}$? $\frac{2}{3}$ of $7\frac{1}{4}$?

15. What is $\frac{3}{4}$ of $2\frac{1}{5}$? $\frac{3}{5}$ of $7\frac{1}{2}$? $\frac{3}{7}$ of $2\frac{1}{2}$?

16. When fractions are connected by the word *of*, what does the *of* denote? **Multiplication.**

WRITTEN EXERCISES.

216.—1. Multiply $\frac{8}{9}$ by $\frac{3}{7}$, or find $\frac{3}{7}$ of $\frac{8}{9}$.

$\frac{8}{9} \times \frac{3}{7} = \frac{24}{63} = \frac{8}{21}$.

Or,

$\frac{8}{9} \times \frac{3}{7} = \frac{8 \times \cancel{3}}{\cancel{9} \times 7} = \frac{8}{21}$

SOLUTION.—$\frac{3}{7}$ of $\frac{8}{9}$ is the same as 3 times $\frac{1}{7}$ of $\frac{8}{9}$; $\frac{1}{7}$ of $\frac{8}{9}$ is $\frac{8}{9 \times 7}$, or $\frac{8}{63}$, and 3 times $\frac{8}{63}$ is $\frac{8 \times 3}{63}$, or $\frac{24}{63}$, which, reduced, is $\frac{8}{21}$, the product required. Or, indicating the multiplication and canceling, we have $\frac{8}{21}$, as before.

2. What is the product of $8\frac{2}{3}$ by $4\frac{1}{5}$?

$8\frac{2}{3} = \frac{26}{3}$; $4\frac{1}{5} = \frac{21}{5}$

$\frac{26}{\cancel{3}} \times \frac{\cancel{21}^{7}}{5} = \frac{182}{5} = 36\frac{2}{5}$

SOLUTION.—Reducing the mixed numbers to equivalent fractions, we have $8\frac{2}{3} \times 4\frac{1}{5} = \frac{26}{3} \times \frac{21}{5}$. Canceling and multiplying, we have $\frac{182}{5}$, which, reduced, is $36\frac{2}{5}$, the product required.

3. Multiply $\frac{3}{17}$ by $\frac{4}{5}$, or find $\frac{4}{5}$ of $\frac{3}{17}$.

4. Multiply $13\frac{5}{8}$ by $2\frac{2}{3}$. $29\frac{1}{11}$ by $6\frac{1}{4}$.

217. **Rules for Multiplying a Fraction by a Fraction.**—*Multiply the numerators together for the numerator, and the denominators for the denominator of the product.*

If there be mixed numbers, reduce them to fractions and then multiply.

PROBLEMS.

218. Multiply—

1. $\frac{5}{18}$ by $\frac{2}{3}$.
2. $\frac{7}{31}$ by $\frac{3}{4}$.
3. $\frac{6}{19}$ by $\frac{5}{6}$.
4. $\frac{21}{108}$ by $\frac{6}{7}$.
5. $\frac{105}{311}$ by $\frac{5}{17}$.
6. $3\frac{5}{16}$ by $\frac{7}{8}$.
7. $16\frac{3}{8}$ by $\frac{2}{3}$.
8. $\frac{31}{42}$ by $3\frac{1}{9}$.
9. $\frac{17}{65}$ by $7\frac{1}{5}$.
10. $\frac{42}{65}$ by $\frac{11}{14}$.
11. 12 by $1\frac{2}{3}$.
12. $17\frac{3}{5}$ by $5\frac{1}{11}$.
13. What is the value of $\frac{7}{8}$ of $\frac{15}{16}$?
14. What is the value of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{24}{27}$?
15. John owns $\frac{3}{4}$ of a boat, and his brother owns $\frac{4}{15}$ as much of it. What part of the boat does his brother own?
16. What will $\frac{7}{8}$ of a yard of cloth cost, at $3\frac{1}{5}$ dollars a yard?

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219. TEST QUESTIONS.—1. In what two ways may a fraction be multiplied? Show that multiplying the numerator multiplies the fraction. That dividing the denominator multiplies the fraction.

2. How is an integer multiplied by a fraction? Show how you can take of a number the part denoted by a fraction. In what two ways may an integer be multiplied by a fraction?

3. How do you multiply a fraction by a fraction? How do you proceed if there are mixed numbers?

4. When fractions are connected by the word of, what does the *of* denote?

SECTION XVIII.

DIVISION OF FRACTIONS.

CASE I.

Fractions Divided by Integers.

220.—1. A boy having $\frac{3}{4}$ of an orange wished to divide it equally among 3 boys. What part of the orange could he give to each?

2. $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1?

3. If 5 books can be bought for $\frac{5}{6}$ of a dollar, what will 1 book cost?

4. If 4 yards of cloth can be bought for $\frac{5}{6}$ of a dollar, what will 1 yard cost?

SOLUTION.—If 4 yards of cloth cost $\frac{5}{6}$ of a dollar, 1 yard will cost $\frac{1}{4}$ of $\frac{5}{6}$ of a dollar, or $\frac{5}{24}$ of a dollar.

5. If 1 man can do a piece of work in $\frac{3}{4}$ of a day, in what time can 5 men do it?

6. $\frac{1}{5}$ of $\frac{3}{4}$ is what part of 1?

7. When 6 books of equal value cost $\frac{12}{5}$ of a dollar, what is the cost of 1 book?

8. $\frac{7}{8}$ divided by 6, or $\frac{1}{6}$ of $\frac{7}{8}$, is what number?

9. Divide $\frac{1}{5}$ by 2. $\frac{6}{7}$ by 3. $\frac{3}{8}$ by 5.

10. Divide $\frac{6}{8}$ by 3. $\frac{4}{9}$ by 4. $\frac{10}{11}$ by 5.

11. Dividing the numerator of $\frac{10}{11}$ by 5 gives $\frac{2}{11}$; multiplying the denominator of $\frac{10}{11}$ by 5 gives $\frac{10}{55}$, which equals $\frac{2}{11}$; in each case the fraction is divided by 5. How, then, may a fraction be divided by an integer?

12. Divide $\frac{16}{5}$ by 8. $\frac{3}{5}$ by 6. $\frac{14}{15}$ by 7.

13. Divide $\frac{45}{63}$ by 3. $\frac{20}{6}$ by 5. $\frac{18}{9}$ by 9.

14. If 1 man can do a piece of work in $8\frac{3}{4}$ days, in what time can 3 men do it?

SOLUTION.—If 1 man can do a piece of work in $8\frac{3}{4}$ days, 3 men can do it in $\frac{1}{3}$ of $8\frac{3}{4}$ days, or in $2\frac{1}{12}$ days.

15. If 4 bushels of corn cost $5\frac{3}{4}$ dollars, what is the cost of 1 bushel?

16. Divide $12\frac{1}{2}$ by 7. $9\frac{3}{5}$ by 6. $10\frac{1}{7}$ by 9.

221. Principle.—*Dividing the numerator or multiplying the denominator by any number divides the fraction by that number.*

WRITTEN EXERCISES.

222.—1. What is the quotient of $\frac{12}{13}$ divided by 6?

$$\frac{12}{13} \div 6 = \frac{12 \div 6}{13} = \frac{2}{13}$$

Or,

$$\frac{12}{13} \div 6 = \frac{\overset{2}{\cancel{12}}}{13 \times \cancel{6}} = \frac{2}{13}$$

SOLUTION.—Since dividing the numerator of a fraction divides the fraction, $\frac{12}{13}$ divided by 6 gives $\frac{2}{13}$, the quotient required.

Or, since multiplying the denominator of a fraction divides the fraction, $\frac{12}{13}$ divided by $6 = \frac{12}{13 \times 6} = \frac{2}{13}$, the same result.

2. What is the quotient of $31\frac{1}{4}$ divided by 5?

$$31\frac{1}{4} \div 5 = \frac{\overset{25}{\cancel{125}}}{4 \times \cancel{5}} = \frac{25}{4} = 6\frac{1}{4}$$

Or,

$$5\overline{)31\frac{1}{4}}$$
$$6\frac{1}{4}$$

SOLUTION.—Since $31\frac{1}{4}$ is $\frac{125}{4}$, we may divide $31\frac{1}{4}$ by 5 by dividing its equivalent, $\frac{125}{4}$, which gives $\frac{25}{4}$, or $6\frac{1}{4}$, the quotient required.

Or, we may divide the mixed number without first reducing it to an improper fraction. One fifth of 31 is 6, with a remainder 1, which is equal to $\frac{4}{4}$; this added to the $\frac{1}{4}$ gives $\frac{5}{4}$. One fifth of $\frac{5}{4}$ is $\frac{1}{4}$, which added to 6 gives $6\frac{1}{4}$, the same result.

3. Divide $\frac{9}{31}$ by 3. $\frac{17}{62}$ by 34. $23\frac{1}{4}$ by 7.

223. Rules for the Division of a Fraction by an Integer.—*Divide the numerator, or multiply the denominator, by the integer.*

If the dividend is a mixed number, reduce it to an improper fraction before dividing; or, divide the integer and fraction separately, and unite the results.

PROBLEMS.

224. Divide—

1. $\frac{7}{8}$ by 6.	4. $\frac{3}{7}$ by 27.	7. $16\frac{1}{2}$ by 7.
2. $\frac{3}{4}$ by 9.	5. $\frac{4}{47}$ by 15.	8. $18\frac{3}{4}$ by 8.
3. $\frac{11}{12}$ by 22.	6. $\frac{14}{19}$ by 7.	9. $25\frac{1}{5}$ by 20.

10. If 11 boys should have $90\frac{6}{25}$ dollars divided equally among them, how much would each receive?

CASE II.

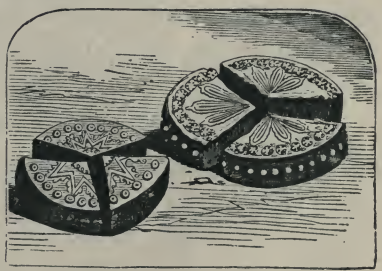
Integers Divided by Fractions.

225.—1. How many thirds of a cake in 1 cake? In 2 cakes?

2. How many times $\frac{2}{3}$ of a cake in 2 cakes?

3. How many pears, at $\frac{2}{3}$ of a cent each, can be bought for 2 cents?

4. When tea is $\frac{4}{5}$ of a dollar a pound, how many pounds can be bought for 6 dollars?



SOLUTION.—If $\frac{4}{5}$ of a dollar will purchase 1 pound of tea, 6 dollars will purchase as many pounds as $\frac{4}{5}$ of a dollar is contained times in 6 dollars. 6 dollars equal $\frac{30}{5}$ of a dollar; $\frac{30}{5} \div \frac{4}{5} = 30 \div 4 = 7\frac{2}{4}$ or $7\frac{1}{2}$. Hence, $7\frac{1}{2}$ pounds can be bought.

5. At $\frac{5}{8}$ of a dollar a yard, how many yards of cloth can you buy for 3 dollars?

6. 1 is how many times $\frac{2}{3}$? $\frac{5}{7}$? $\frac{4}{5}$?

7. 2 are how many times $\frac{3}{4}$? $\frac{8}{9}$? $\frac{3}{8}$?

8. 8 are how many times $\frac{2}{3}$? $\frac{3}{4}$? $\frac{7}{8}$?

WRITTEN EXERCISES.

226.—1. What is the quotient of 15 divided by $\frac{3}{4}$?

$15 \div \frac{3}{4} = \frac{60}{4} \div \frac{3}{4} = 20$

Or,

$15 \div \frac{3}{4} = \frac{\overset{5}{\cancel{15}} \times 4}{\cancel{3}} = 20$

SOLUTION.—15 is equal to $\frac{60}{4}$. 60 fourths divided by 3 fourths gives 20, the quotient required.

Or, since $15 \div 1$ is 15, $15 \div \frac{1}{4}$ must be 4 times 15, and $15 \div \frac{3}{4}$ must be $\frac{1}{3}$ of 4 times 15, which is 20, the same result.

2. What is the quotient of 64 divided by $\frac{7}{8}$?

3. Divide 25 by $\frac{3}{4}$. 112 by $\frac{5}{6}$. 98 by $\frac{4}{9}$.

227. Rules for Division of an Integer by a Fraction.—*Reduce the integer to a fraction having the same denominator as the divisor, and divide the numerator of the dividend by the numerator of the divisor. Or. Multiply the integer by the denominator of the divisor, and divide the result by the numerator.*

PROBLEMS.

228. Divide—

1. 15 by $\frac{4}{7}$.	5. 51 by $\frac{1}{17}$.	9. 24 by $\frac{2}{7}$.
2. 61 by $\frac{2}{3}$.	6. 43 by $\frac{7}{8}$.	10. 17 by $\frac{5}{12}$.
3. 21 by $\frac{2}{5}$.	7. 65 by $\frac{3}{11}$.	11. 36 by $\frac{4}{9}$.
4. 40 by $\frac{5}{8}$.	8. 90 by $\frac{4}{15}$.	12. 110 by $\frac{11}{20}$.

13. What is the quotient of 40 divided by $3\frac{1}{8}$?

SOLUTION.

$3\frac{1}{8} = \frac{25}{8}$, and $40 = \frac{320}{8}$; $320 \div 25 = 12\frac{20}{25} = 12\frac{4}{5}$

14. Divide 17 by $2\frac{5}{8}$. 28 by $1\frac{4}{7}$. 42 by $6\frac{2}{3}$.

15. At $\frac{7}{8}$ of a dollar a yard, how many yards of cloth can be purchased for $8\frac{3}{4}$ dollars?

CASE III.

Fractions Divided by Fractions.

229.—1. At $\frac{1}{4}$ of a dollar a yard, how much cloth can be bought for $\frac{3}{4}$ of a dollar?

2. At $\frac{1}{2}$ of a dollar a bushel, how many bushels of apples can be bought for $\frac{3}{4}$ of a dollar?

3. How many times is $\frac{2}{5}$ contained in $\frac{3}{4}$?

SOLUTION.—$\frac{3}{4} \div \frac{2}{5}$ is equivalent to $\frac{15}{20}$ divided by $\frac{8}{20}$, and 8 twentieths are contained in 15 twentieths $1\frac{7}{8}$ times, which is the result required.

4. Divide $\frac{7}{8}$ by $\frac{2}{5}$. $\frac{3}{4}$ by $\frac{2}{3}$. $\frac{7}{9}$ by $\frac{2}{7}$. $\frac{7}{9}$ by $\frac{1}{6}$.

5. At $\frac{3}{10}$ of a dollar a pound, how many pounds of sugar can be bought for $\frac{7}{8}$ of a dollar?

6. At $\frac{3}{10}$ of a dollar each, how many books can be bought for $2\frac{2}{5}$ dollars?

SOLUTION.—$2\frac{2}{5}$ is equal to $\frac{24}{10}$; at $\frac{3}{10}$ of a dollar each, as many books can be bought for $\frac{24}{10}$ of a dollar as 3 tenths are contained times in 24 tenths, or 8, which is the number required.

7. When butter is $\frac{1}{3}$ of a dollar a pound, how many pounds can you buy for $2\frac{3}{4}$ dollars?

DEFINITION.

230. A **Complex Fraction** is one that has a fraction in one or both of its terms.

Thus, $\frac{\frac{3}{4}}{\frac{2}{5}}$ is a complex fraction, and indicates the division of $\frac{3}{4}$ by $\frac{2}{5}$.

WRITTEN EXERCISES.

231.—1. What is the quotient of $\frac{4}{5}$ divided by $\frac{2}{9}$?

$\frac{4}{5} \div \frac{2}{9} = \frac{36}{45} \div \frac{10}{45} = \frac{36}{10} = \frac{18}{5} = 3\frac{3}{5}$

SOLUTION.—$\frac{4}{5}$ and $\frac{2}{9}$ reduced to fractions having a common denominator are $\frac{36}{45}$ and $\frac{10}{45}$; we can then divide $\frac{4}{5}$ by $\frac{2}{9}$, by finding the quotient of 36 forty-fifths divided by 10 forty-fifths, which is $3\frac{3}{5}$, the quotient required. Or,

$\frac{4}{5} \div \frac{2}{9} = \frac{4 \times 9}{5 \times 2} = \frac{18}{5} = 3\frac{3}{5}$

Since $\frac{4}{5} \div 1$ is $\frac{4}{5}$, $\frac{4}{5} \div \frac{1}{9}$ must be 9 times $\frac{4}{5}$, or $\frac{4 \times 9}{5}$, and $\frac{4}{5} \div \frac{2}{9}$ must be $\frac{1}{2}$ of 9 times $\frac{4}{5}$, or $\frac{4 \times 9}{5 \times 2}$, which, reduced, is $3\frac{3}{5}$, the same result.

2. What is the quotient of $\frac{11}{12}$ divided by $\frac{2}{3}$?

3. What is the quotient of $\frac{7}{9}$ divided by $\frac{2}{21}$?

232. Rules for Division of a Fraction by a Fraction.—*Reduce the fractions, if necessary, to a common denominator, and divide the numerator of the dividend by the numerator of the divisor. Or,*

Multiply the dividend by the denominator of the divisor, and divide the result by the numerator.

PROBLEMS.

233. Divide—

1. $\frac{6}{7}$ by $\frac{3}{14}$.
2. $\frac{11}{12}$ by $\frac{5}{6}$.
3. $\frac{15}{16}$ by $\frac{7}{8}$.
4. $\frac{17}{18}$ by $\frac{5}{9}$.
5. $\frac{4}{5}$ by $\frac{2}{7}$.
6. $\frac{5}{11}$ by $\frac{2}{3}$.
7. $\frac{7}{9}$ by $\frac{3}{4}$.
8. $\frac{10}{13}$ by $\frac{1}{52}$.
9. $\frac{1}{20}$ by $\frac{7}{10}$.
10. $\frac{5}{19}$ by $\frac{2}{38}$.
11. $\frac{6}{23}$ by $\frac{1}{2}$.
12. $\frac{3}{22}$ by $\frac{6}{11}$.

13. What is the quotient of $3\frac{1}{8}$ divided by $1\frac{1}{4}$?

SOLUTION.

$3\frac{1}{8} = \frac{25}{8}$, and $1\frac{1}{4} = \frac{5}{4}$; $\frac{25}{8} \times \frac{4}{5} = \frac{5}{2} = 2\frac{1}{2}$

14. How many barrels of potatoes, at $2\frac{3}{4}$ dollars a barrel, can be bought for $16\frac{1}{2}$ dollars?

15. How many yards of calico, at $\frac{3}{16}$ of a dollar a yard, can be bought for $\frac{7}{8}$ of a dollar?

16. What is the value of $\frac{\frac{7}{8}}{\frac{3}{16}}$? Of $2\frac{\frac{5}{6}}{\frac{2}{3}}$?

17. How many tons of coal, at $3\frac{3}{4}$ dollars a ton, can be bought for $13\frac{1}{8}$ dollars?

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234. TEST QUESTIONS.—1. In what two ways can a fraction be divided by an integer? If the dividend be a mixed number, how do you divide?

2. How do you divide a fraction by an integer? An integer by a fraction?

3. What is a complex fraction? Express in the fractional form, the division of some fraction by a fraction. How do you divide a fraction by a fraction?

SECTION XIX.

REVIEW OF FRACTIONS.

235.—1. How many fifths in $4\frac{2}{5}$? In $7\frac{3}{5}$? In $9\frac{4}{5}$?

2. How many ones in $\frac{2}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?

3. What is the value of $\frac{19}{6}$? Of $\frac{43}{8}$?

4. What improper fraction is equal to $6\frac{7}{9}$?

5. Express $\frac{6}{8}$ in its lowest terms.

6. Express $\frac{3}{4}$ in higher terms.

7. Reduce $\frac{2}{3}$ and $\frac{5}{6}$ each to eighteenths.

8. Reduce $\frac{7}{8}$ and $\frac{1}{5}$ to fractions having a common denominator.

9. A sum of money, diminished by $\frac{2}{5}$ of a dollar, is equal to $2\frac{7}{20}$ dollars. What is the sum?

10. If I sell an article for $\frac{4}{5}$ of its cost, what fractional part of the cost do I lose?

11. If I sell an article for $\frac{1}{4}$ part more than its cost, for how many fourths of the cost do I sell it?

12. If I pay $37\frac{1}{2}$ cents for a knife, and sell it for $\frac{4}{6}$ of its cost, how many cents do I lose?

13. A farmer had $11\frac{1}{2}$ bushels of wheat stolen, which was $\frac{1}{6}$ of all he had. How much had he?

14. $11\frac{1}{2}$ is $\frac{1}{6}$ of what number?

15. If a slate cost $\frac{7}{20}$ of a dollar, how many slates can be bought for $1\frac{3}{4}$ dollars?

WRITTEN EXERCISES.

236.—1. Reduce $105\frac{3}{8}$ to eighths.

2. Reduce $\frac{7}{18}$ and $\frac{5}{12}$ to fractions having the least common denominator.

3. Reduce $\frac{42}{63}$ and $\frac{84}{108}$ each to its lowest terms.

4. Having lost $\frac{5}{8}$ of a dollar, I find I have left $13\frac{3}{4}$ dollars. What sum had I at first?

5. A merchant owned $\frac{48}{64}$ of a ship, and sold $\frac{3}{8}$ of the ship. What part of the ship had he left?

6. $5\frac{1}{4}$ is $\frac{1}{5}$ of what number? $\frac{1}{6}$ of what number?

7. $\frac{13}{41}$ is $\frac{1}{7}$ of what number? $\frac{1}{8}$ of what number?

8. $\frac{11}{19}$ is $\frac{1}{9}$ of what number? $\frac{1}{10}$ of what number?

9. If a horse will eat, in a given time, $\frac{17}{20}$ of a ton of hay; a cow, $\frac{3}{4}$ of a ton; and an ox, $\frac{9}{10}$ of a ton, what quantity will they all eat in the same time?

10. A lady gave $10\frac{3}{4}$ dollars for a dress, $4\frac{1}{2}$ dollars for a shawl, and $\frac{7}{8}$ of a dollar for a handkerchief. How much did they all cost her?

11. James gave $\frac{1}{15}$ of his money for a coat, and $\frac{5}{12}$ of it for a library. What part of it had he left?

12. What will 19 pounds of tea cost, at $1\frac{3}{8}$ dollars a pound?

13. The multiplicand is $12\frac{2}{3}$ and the multiplier $3\frac{5}{9}$. What is the product?

14. Smith owns $\frac{2}{3}$ of $\frac{5}{3\frac{1}{2}}$ of a ship, and Collins owns $\frac{4}{5}$ as much as Smith. What part does Collins own?

MENTAL EXERCISES.

237.—1. 20 is $\frac{2}{7}$ of what number?

SOLUTION.—If 20 is $\frac{2}{7}$ of some number, $\frac{1}{7}$ of that number must be $\frac{1}{2}$ of 20, or 10, and $\frac{7}{7}$, or the whole, of the number, must be 7 times 10, which is 70, the number required.

2. 40 is $\frac{5}{8}$ of what number? $\frac{2}{3}$ of what number?

3. 30 is $\frac{2}{3}$ of what number? $\frac{5}{6}$ of what number?

4. 16 is $\frac{4}{5}$ of what number? $\frac{3}{8}$ of what number?

5. 20 is $\frac{5}{9}$ of what number? $\frac{4}{7}$ of what number?

6. If $\frac{5}{8}$ of an acre of land be worth 40 dollars, what is an acre worth?

7. A farmer sold a cow at a gain of 20 dollars, which was $\frac{2}{7}$ of her cost. What was the cost?

8. Jane is 16 years old, and is $\frac{4}{5}$ as old as her sister. What is the age of her sister?

9. A pole stands $\frac{2}{7}$ of its length in the mud, $\frac{3}{7}$ in the water, and the remainder, which is 5 feet, above water. What is the length of the pole?

10. When $\frac{3}{4}$ of a dollar is $\frac{2}{3}$ of the price of a pound of tea, what is the price of a pound?

SOLUTION.—If $\frac{3}{4}$ of a dollar is $\frac{2}{3}$ of the price of a pound, $\frac{1}{3}$ of the price must be $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$, of a dollar, and $\frac{3}{3}$, or the whole price, must be 3 times $\frac{3}{8}$, or $\frac{9}{8}$, equal to $1\frac{1}{8}$ dollars.

11. $\frac{3}{4}$ is $\frac{2}{3}$ of what number? $\frac{3}{4}$ of what number?

12. If $\frac{2}{5}$ of a rod is $\frac{5}{6}$ of the width of a walk, what is the width of the walk?

13. $\frac{2}{5}$ is $\frac{5}{6}$ of what number? $\frac{2}{3}$ of what number?

14. How many times $\frac{2}{5}$ is $\frac{7}{4}$? How many times $\frac{2}{7}$ is $\frac{2}{3}$?

15. If 3 men can do a piece of work in $4\frac{1}{3}$ days, in how many days can 5 men do it?

16. $4\frac{1}{3}$ is $\frac{5}{3}$ of what number? $\frac{5}{6}$ of what number?

17. How many yards of cloth, 3 quarters of a yard wide, are equal to 7 yards 5 quarters wide?

18. 7 is $\frac{3}{5}$ of what number? $\frac{3}{7}$ of what number?

19. 12 is $\frac{3}{4}$ of how many times 2?

20. 30 is $\frac{5}{6}$ of how many times 12?

21. $\frac{3}{5}$ of 10 is $\frac{2}{3}$ of what number?

22. $\frac{3}{4}$ of 12 is $\frac{3}{8}$ of how many times 4?

23. If $\frac{4}{5}$ of a barrel of beef cost 16 dollars, how many cords of wood at 5 dollars a cord will pay for a barrel of beef?

24. If $\frac{3}{5}$ of a ton of coal cost 4 dollars, what will $\frac{2}{3}$ of a ton cost?

25. If $\frac{3}{4}$ of a yard of cloth cost 3 dollars, what will $\frac{5}{8}$ of a yard cost?

WRITTEN EXERCISES.

238.—1. At $\frac{5}{8}$ of a dollar a yard, how many yards of cloth can be bought for $16\frac{1}{4}$ dollars?

2. Divide $\frac{438}{27}$ by 18. $\frac{11}{64}$ by 32. $3\frac{1}{3}$ by $\frac{5}{6}$.

3. I paid $95\frac{7}{10}$ dollars for flour, at the rate of $8\frac{7}{10}$ dollars per barrel. How many barrels did I buy?

4. If $\frac{4}{7}$ of an acre of land cost 96 dollars, what will $\frac{3}{4}$ of an acre cost?

5. If the divisor is $\frac{6}{7}$ and the quotient $5\frac{3}{5}$, what is the dividend?

6. If $\frac{3}{4}$ of a ton of hay cost 18 dollars, what will $\frac{1}{2}\frac{3}{0}$ of a ton cost?

7. If the product of two factors is $15\frac{8}{9}$, and one of the factors is $3\frac{1}{4}$, what is the other factor?

8. If the divisor is $\frac{5}{2}\frac{}{7}$, and the dividend is $\frac{2}{4}\frac{0}{5}\frac{}{9}$, what is the quotient?

9. I own $\frac{11}{16}$ of a ship, and my brother owns $\frac{3}{6}\frac{}{4}$. How many times as much as he, do I own?

10. John sold 320 acres of land; he then bought $\frac{3}{4}$ as many as he sold, and found that number to be $\frac{3}{8}$ as many as he had at first. How many had he at first?

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239. TEST QUESTIONS.—1. What is an integer? A fraction? A mixed number?

2. What are the terms of a fraction? Which term is the numerator? Which the denominator? How is a fraction expressed by figures?

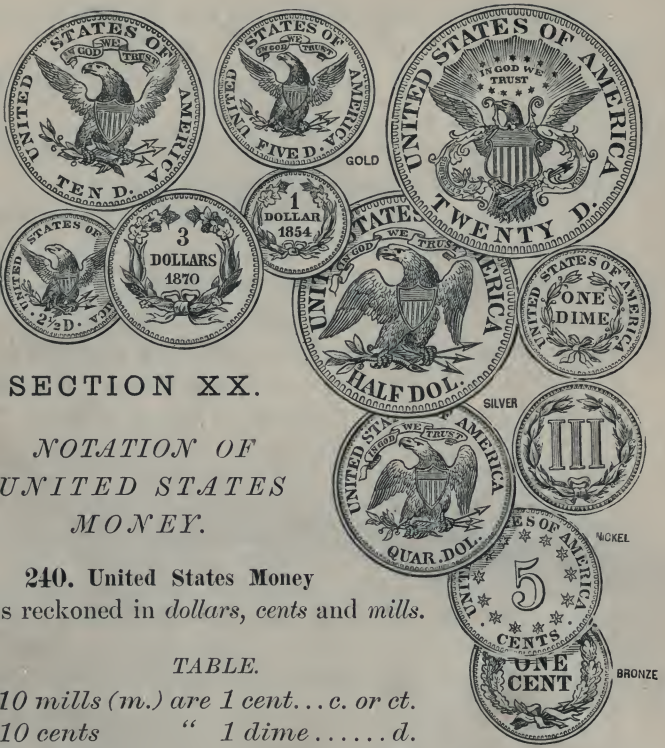
3. What is a proper fraction? An improper fraction? A complex fraction?

4. What is the value of a fraction? Is the value of a fraction changed by multiplying both terms by the same number? By dividing both terms by the same number?

5. What is reduction of fractions? How may fractions be changed to fractions with higher terms? To fractions having lower terms?

6. By what means may fractions be added or subtracted? How are fractions with different denominators prepared for addition or subtraction?

7. What is a principle in addition of fractions? In subtraction of fractions? In multiplication of fractions? In division of fractions?



SECTION XX.

NOTATION OF UNITED STATES MONEY.

240. United States Money is reckoned in *dollars, cents* and *mills.*

TABLE.

10 mills (m.) are 1 cent...c. or ct.
10 cents " 1 dime.....d.
10 dimes " 1 dollar.....$.

$1=10d.=100ct.=1000m.

241. United States Money consists of **Coins** and **Paper Money.**

242. Coins are pieces of metal stamped for use as money.

The coins of the United States are now made of gold, silver, nickel or bronze.

TABLE OF COINS.

	Names.	Values.		Names.	Values.
Gold.	Double-eagle	$20.	**Silver.**	Dollar	100c.
	Eagle	10.		Half-dollar	50c.
	Half-eagle	5.		Quarter-dollar	25c.
	3-dollar piece	3.		20-cent piece	20c.
	Quarter-eagle	2½.		10-cent piece	10c.
	Dollar	1.	**Nickel.**	5-cent piece	5c.
Bronze.	Cent	1c.		3-cent piece	3c.

In addition to the above, a *Trade-Dollar* is issued for the convenience of foreign trade.

243. Paper Money consists of Notes, issued by banks and by the Treasury of the United States, as substitutes for coin. When Treasury Notes are of less face-value than $1, they are called *Fractional Currency.*

244. In business transactions dimes are regarded as a number of cents.

Thus, 5 dimes are regarded as 50 cents.

In denoting by figures sums of United States money, the sign $ is placed before dollars, and the decimal point (.) is placed before cents.

Thus, $5.90 expresses five dollars and ninety cents.
$.075 expresses seven cents and five mills.

245.—1. How many mills in 1 cent? In 3 cents? In 9 cents?

2. How many cents in 10 mills? In 30 mills? In 90 mills?

3. How many cents in 1 dime? In 5 dimes? In 8 dimes?

4. How many dimes in 10 cents? In 50 cents? In 80 cents?

5. How many dimes in 1 dollar? In 5 dollars?

6. How many dollars in 10 dimes? In 50 dimes?

7. How many cents in 10 dimes? In 1 dollar? In 2 dollars? In 5 dollars?

8. How many dimes, and how many cents over, in 25 cents? In 42 cents? In 85 cents?

9. How many dollars, and how many cents over, in 125 cents? In 250 cents? In 375 cents?

10. How many cents, and how many mills over, in 15 mills? In 45 mills? In 67 mills?

11. Since 10 mills are 1 cent, what part of a cent is 1 mill? Is 2 mills? Is 7 mills?

12. Since 10 cents are 1 dime, what part of a dime is 1 cent? Is 3 cents? Is 9 cents?

13. Since 10 dimes are 1 dollar, what part of a dollar is 1 dime? Is 2 dimes? Is 7 dimes?

14. Since 100 cents are 1 dollar, what part of a dollar is 1 cent? Is 3 cents? Is 17 cents?

15. Since 1000 mills are 1 dollar, what part of a dollar is 1 mill? Is 7 mills? Is 117 mills?

WRITTEN EXERCISES.

246. Copy and read—

1. \$15.25.	4. \$143.41.	7. \$3.057.	10. \$16.15.
2. \$7.08.	5. \$205.75.	8. \$1.305.	11. \$11.31.
3. \$97.375.	6. \$97.334.	9. \$41.065.	12. \$97.005.

247. Write in figures—

1. Ten dollars sixteen cents.
2. Twenty-seven dollars five cents.
3. Sixty-three dollars thirty-one cents.
4. Five hundred seventeen dollars.

SECTION XXI.

REDUCTION OF UNITED STATES MONEY.

248.—1. Change \$7 to cents and to mills.

7 = *No. of dollars.*
100
700 = *No. of cents.*
10
7000 = *No. of mills.*

SOLUTION.—Since \$1 is equal to 100 cents, \$7 must equal 7 times 100 cents, or 700 cents.

Since 1 cent is equal to 10 mills, 700 cents must equal 700 times 10 mills, or 7000 mills.

2. Change \$15 to cents and to mills.
3. Change \$15.67 to cents.

\$15.67 = *1567 cents.*

SOLUTION.—\$15.67 may be changed to cents by removing the dollar-sign and decimal point, which gives 1567 cents, because \$15 = 1500 cents, and 1500 cents plus 67 cents are 1567 cents.

4. Change \$37.03 to cents.
5. Change \$43.444 to mills.

\$43.444 = *43444 mills.*

SOLUTION.—\$43.444 may be changed to mills by removing the dollar-sign and decimal point, which gives 43444 mills; because \$43 = 43000 mills, 44 cents = 440 mills, and 43000 mills, plus 440 mills, plus 4 mills, are 43444 mills.

6. Change \$6.305 to mills.
7. Change 7000 mills to cents and to dollars.

7000 = *No. of mills.*
700 = *No. of cents.*
7 = *No. of dollars.*

SOLUTION.—Since there must be one tenth as many cents as there are mills, 7000 mills may be changed to cents by dividing by 10, or by removing one cipher from the right, which gives 700 as the number of cents.

Since there must be one hundredth as many dollars as there are cents, 700 cents may be changed to dollars by dividing by 100, or by removing two ciphers from the right, which gives 7 as the number of dollars.

8. Change 93000 mills to cents and to dollars.

249. Rules for Reduction of United States Money.—*To reduce dollars to cents, remove the dollar-sign and annex two ciphers; to reduce dollars to mills, annex three ciphers; to reduce cents to mills, annex one cipher.*

To reduce dollars and cents to cents, or dollars, cents and mills to mills, remove the dollar-sign and the decimal point.

To reduce cents to dollars, point off two orders from the right and prefix the dollar-sign; to reduce mills to dollars, point off three orders from the right and prefix the dollar-sign; and to reduce mills to cents, point off one order from the right and prefix the dollar-sign.

PROBLEMS.

250. Reduce—

1. $57 to cents.
2. $53 to mills.
3. 98 cents to mills.
4. 5700 cents to dollars.
5. 53000 mills to dollars.
6. 980 mills to cents.

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251. TEST QUESTIONS.—1. In what is United States money reckoned? Of what does it consist? Recite the table.

2. What are coins? Of what are the coins of United States money made? Name the coins made of gold. Of silver. Of nickel. Of bronze. What is Paper Money? What is Fractional Currency?

3. How are dimes regarded in business? In denoting sums of money by figures, what sign denotes dollars? What point is placed before cents?

SECTION XXII.

COMPUTATIONS IN UNITED STATES MONEY.

ADDITION.

252.—1. If you should pay 30 cents for a slate, 25 cents for a writing-book and 10 cents for a pencil, how much would you pay for all?

2. How much is 30 cents + 25 cents + 10 cents?

3. Susan gave 50 cents for a collar, 40 cents for a thimble and 12 cents for some needles. How much did she give for the whole?

4. If you give $9.50 for a coat and $3.25 for a vest, how much do you give for both?

WRITTEN EXERCISES.

253.—1. What is the sum of $95.60, $19 and $4.375?

$$\begin{array}{r} \$95.60 \\ 19. \\ 4.375 \\ \hline \$118.975 \end{array}$$

SOLUTION.—Write the numbers and add, as required by the rule for addition, and separate the dollars from the cents in the sum by a decimal point.

Write and add—

(2.)	(3.)	(4.)	(5.)
$45.13	$2.375	$11.14	$3.72
5.07	6.25	63.15	.144
17.	7.625	7.99	5.138
$67.20			$9.002

6. $144.56 + $17.18 + $100.63 = what amount?

7. Johnson paid for a farm $6500, and for improvements on it $365.50. He then sold it for $150 more than the whole cost; for how much did he sell it?

SUBTRACTION.

254.—1. Arthur had 95 cents, and gave 50 cents for a knife. How much had he left?

2. 85 cents less 17 cents are how many cents?

3. If you had $1.25, and should give 75 cents for an arithmetic, how much would you have left?

4. What sum added to 75 cents will make $1.25? What sum added to 50 cents will make $1.25?

5. If I have $9.25, how much more must I get to pay for a coat worth $10?

6. I have 7 dimes; how much more must I get to pay for a book worth $1.10?

WRITTEN EXERCISES.

255.—1. What is the difference between $106 and $43.50?

$$\begin{array}{r} \$106.00 \\ 43.50 \\ \hline \$62.50 \end{array}$$

SOLUTION.—Write the numbers and subtract, as required by the rule for subtraction, and separate the dollars from the cents in the difference by a decimal point.

Write and subtract—

(2.)	(3.)	(4.)	(5.)
$164.15	$115.000	$31.15	$347.00
87.09	37.085	4.17	243.19

6. How much less is $1867.25 than $5555.43?

7. If you purchase goods at a cost of $316.50, and sell them for $400, how much do you gain?

MULTIPLICATION.

256.—1. At 12 cents each, what will 12 writing-books cost?

2. At 25 cents each, what will 5 spelling-books cost?

3. James paid \$1.25 for a hat, and Edward paid 3 times as much. How much did Edward pay?

4. If flour is \$10 a barrel, what will 10 barrels cost?

5. What will 5 bushels of wheat cost, at \$2.10 a bushel?

WRITTEN EXERCISES.

257.—1. What is the product of \$109.50 multiplied by 9?

\$109.50
9
\$985.50

SOLUTION.—Write the numbers and multiply, as required by the rule for multiplication, and separate the dollars from the cents in the product by a decimal point.

Multiply—

2. \$61.34 by 8.
3. \$40.65 by 11.
4. \$3.125 by 25.
5. \$19.06 by 12.
6. \$20.013 by 13.
7. \$93.56 by 100.
8. \$1005 by 13.
9. \$1056 by 171.
10. \$34.055 by 1000.
11. \$103.03 by 100.

12. \$13.06 × 10 × 3 = what amount?

13. What will it cost to build 7 cottages, at \$2500.50 each?

14. If a man earns \$125.87 each month, how much does he earn in 12 months?

15. How much will 640 acres of land cost, at \$120 per acre?

DIVISION.

258.—1. How many writing-books, at 12 cents each, can be bought for $1.44?

2. At 25 cents each, how many spelling-books can be bought for $1.25?

3. James paid $3.75 for 3 hats, how much were they apiece?

4. At 3 dimes a yard, how many yards of cloth can be bought for $2.10?

WRITTEN EXERCISES.

259.—1. What is the quotient of $985.50 divided by 9?

$$9)\$985.50\ (\$109.50$$

SOLUTION.—Write the numbers and divide, as required by the rule in division, and separate the dollars from the cents in the quotient by a decimal point.

2. What is the cost of a barrel of flour when 14 barrels cost $91?

$$\begin{array}{l} 14)\$91.00(\$6.50 \\ \quad 84 \\ \quad \overline{70} \\ \quad 70 \\ \quad \overline{0} \end{array}$$

SOLUTION.—Continue the division after dividing the dollars, supplying the orders of cents in the dividend by ciphers.

Divide—

3. $325.20 by 8.
4. $67.10 by 11.
5. $626.50 by 25.
6. $73.44 by 12.
7. $9356 by 100.
8. $13065 by 13.
9. $14.56 by 4.
10. $634055 by 1000.
11. $1135 by 20.
12. $19.65 by 15.
13. $114.24 by 6.
14. $111144 by 100.

15. If 11 tons of hay cost $184.25, what is the cost of 1 ton?

16. At $6.50 a barrel, how many barrels of flour can be bought for $91?

650)9100(14
650
2600
2600

SOLUTION.—Prepare the numbers for dividing by reducing both to cents. $6.50 = 650 cents; and $91 = 9100 cents; 9100 ÷ 650 = 14, the result required.

17. How many tons of coal, at $5.25 a ton, can be bought for $105?

18. When 20 tons of coal can be bought for $105, what is the cost of 1 ton?

19. At 80 cents a bushel, how many bushels of corn can be bought for $68?

20. Jones paid for his farm $10564, and Smith bought some land at one-fourth of that sum. What was the cost of Smith's land?

21. One half of Jones's farm is 320 acres; if the whole cost him $7680, what was the price per acre?

ALIQUOT PARTS.

260.—1. What part of a dollar is 50 cents?

2. What part of a dollar is 25 cents?

3. How many cents in half a dollar? In a fourth of a dollar?

4. What part of a dollar is obtained by dividing it by 2? By dividing it by 4?

5. What part of a dollar is 20 cents? Is 10 cents?

6. How many cents in a fifth of a dollar? In a tenth of a dollar?

7. What part of a dollar is obtained by dividing it by 5? By dividing it by 10?

8. What part of a dollar is $33\frac{1}{3}$ cents? $16\frac{2}{3}$ cents? $8\frac{1}{3}$ cents?

9. How many cents in one third of a dollar? In one sixth of a dollar? In one seventh of a dollar? In one twelfth of a dollar?

10. What part of a dollar is obtained by dividing it by 3? By dividing it by 6? By dividing it by 8?

11. What part of a dollar is $12\frac{1}{2}$ cents? Is $37\frac{1}{2}$ cents? Is $62\frac{1}{2}$ cents? Is $87\frac{1}{2}$ cents?

12. How many cents in one eighth of a dollar? In five eighths? In seven eighths?

13. What part of a dollar is $66\frac{2}{3}$ cents? Is 75 cents? Is $83\frac{1}{3}$ cents?

14. What will 25 yards of cloth cost at $12\frac{1}{2}$ cents a yard?

SOLUTION.—If 1 yard cost $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of a dollar, 25 yards will cost 25 times $\frac{1}{8}$ of a dollar, or $\frac{25}{8}$ of a dollar, which are $\$3\frac{1}{8}$, or $\$3.12\frac{1}{2}$.

15. What will 60 pairs of hose cost at 25 cents a pair?

16. What will 37 bushels of apples cost at $33\frac{1}{3}$ cents a bushel?

17. How much must be paid for 30 bushels of corn at $66\frac{2}{3}$ cents a bushel?

SOLUTION.—If $66\frac{2}{3}$ cents, or $\frac{2}{3}$ of a dollar, must be paid for 1 bushel, there must be paid for 30 bushels, 30 times $\frac{2}{3}$ of a dollar, or $\frac{60}{3}$ of a dollar, which are $20.

18. How much must be paid for 16 baskets of peaches at $87\frac{1}{2}$ cents a basket?

19. At 75 cents each, what will 48 arithmetics cost?

DEFINITION.

261. An **Aliquot Part** of a number is any exact fractional part of that number.

In business, frequent use is made of the convenient aliquot parts of a dollar, given in the following—

TABLE.

10 cents are $\frac{1}{10}$ *of \$1.*	$8\frac{1}{3}$ *cents are* $\frac{1}{12}$ *of \$1.*
*20 " " * $\frac{1}{5}$ *of \$1.*	$12\frac{1}{2}$ *" " * $\frac{1}{8}$ *of \$1.*
*25 " " * $\frac{1}{4}$ *of \$1.*	$16\frac{2}{3}$ *" " * $\frac{1}{6}$ *of \$1.*
*50 " " * $\frac{1}{2}$ *of \$1.*	$33\frac{1}{3}$ *" " * $\frac{1}{3}$ *of \$1.*

WRITTEN EXERCISES.

262.—1. What is the cost of 49 yards of cloth at \$1.75 a yard?

SOLUTION.

At \$1.00 per yard, the cost of 49 yards is \$49.
" .50 " " " " " $\frac{1}{2}$ of \$49 = 24.50
" .25 " " " " " $\frac{1}{4}$ of \$49 = 12.25
" \$1.75 " " " " " \$85.75

2. How much must be paid for 54 bushels of wheat at \$1.66$\frac{2}{3}$ per bushel?

3. What will 72 hats cost at 87$\frac{1}{2}$ cents each?

4. What will 96 pounds of rice cost at 8$\frac{1}{3}$ cents a pound?

263. Rule for Finding the Cost of Articles by Aliquot Parts.—*First find the cost at \$1, and then take the aliquot parts of this amount.*

PROBLEMS.

264.—1. What is the value of 758 yards of carpeting at $1.25 per yard?

2. How much must be paid for 178 yards of gingham at $37\frac{1}{2}$ cents per yard?

3. What will 1840 tons of coal cost at $\$4.62\frac{1}{2}$ per ton?

4. At $33\frac{1}{3}$ cents a pair, what is the cost of 96 pairs of gloves?

BILLS.

265. A **Bill of Goods** is a written statement of articles sold, the quantity and price of each article, and the entire cost of the whole.

266. A **Bill of Services** is a written statement of labor performed, the time, kind and value of such services.

267. A **Debtor** is the party who owes a bill, and a **Creditor** is the party to whom the bill is owed.

268. A bill is **Receipted** when the creditor, or some one acting for him, acknowledges its payment in writing.

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269. TEST QUESTIONS.—1. How do you write United States money for adding? For subtracting?

2. How do you write the numbers and multiply in United States money? How do you write the numbers and divide?

3. In dividing dollars, when there is a remainder, how may you continue the division? When the divisor expresses cents and the dividend dollars, how do you prepare the numbers for dividing?

4. What are aliquot parts of a number? How do you compute the cost of articles by aliquot parts?

5. What is a bill of goods? A bill of services? When is a bill receipted?

WRITTEN EXERCISES.

270. Copy the following bills, and compute the amount due—

Bill Unreceipted.

PHILADELPHIA, *Jan.* 4, 1871.

JOHN W. BREWSTER,

Bought of WILLIAM COLLINS & Co.

18 yards of Muslin,	@	$\$.16\frac{2}{3}$	$ 3	00
28 yards of Cambric,	@	.25	7	00
12 dozen Napkins,	@	$2.12\frac{1}{2}$	25	50
6 dozen Towels,	@	$5.25	31	50
			$67	00

The character @ signifies *at.* Thus, 18 yards of muslin @ $\$.16\frac{2}{3}$ means 18 yards of muslin at $\$.16\frac{2}{3}$ per yard.

Bill Receipted.

PORTLAND, *April* 3, 1871.

HENRY C. WARREN,

Bought of HAMSTEAD BROTHERS.

10 pounds Oolong Tea,	@	$1.20	$	
20 pounds Rio Coffee,	@	$.37\frac{1}{2}$		
15 pounds Sugar,	@	$.12\frac{1}{2}$		
			$	

Received payment,

HAMSTEAD BROTHERS.

Bill Receipted by Clerk.

HARTFORD, *March* 11, 1871.

COL. AMBROSE CHASE,

To WILLIAM T. STONE, *Dr.*

1871.				
Feb.	*3*	*For Labor on wall, 9 days, @ $3.50*	*$31*	*50*
"	*10*	" *Excavating cellar,*	*15*	*50*
March	*5*	" *Labor, laying stone, 8 days, @ $3.25,*	*26*	*00*
			$73	*00*

Received payment,

WILLIAM T. STONE,

per M. T. SNEIDER.

Bill with Credit Items.

PROVIDENCE, *April* 27, 1871.

Mr. WALTER BOWEN,

To JOHN BURGESS & SONS, *Dr.*

1871.				
Jan.	*17*	*To 72 tons Coal, @ $8.62½*	*$621*	*00*
April	*8*	" *15 cords Oak Wood, @ 7.25*	*108*	*75*
"	*25*	" *18 cords Pine Wood, @ 6.33⅓*	*114*	*00*
			$843	*75*
		Cr.		
Feb.	*17*	*By Merchandise, as by his bill,* *$550.50*		
"	*24*	*By Cash,* *200.00*	*750*	*50*
		Balance due J. B. & Sons.....	*$ 93*	*25*

SECTION XXIII.

MEASURES OF EXTENSION.

LINEAR MEASURES.

271. **Linear or Long Measures** are those used in ascertaining distances and the dimensions of things.

The units of length are an *inch*, a *yard*, a *rod* and a *mile*.

One Inch.

TABLE.

12 inches (in.)	*are 1 foot*	*..... ft.*
3 feet	*" 1 yard*	*.... yd.*
$5\frac{1}{2}$ yards	*" 1 rod*	*..... rd.*
320 rods	*" 1 mile*	*... mi.*

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

Also, 4 inches are *1 hand*, used in measuring the height of horses; 3 feet are *1 pace;* and 6 feet are *1 fathom*, used in measuring depth of water.

272. In **Cloth Measure** the yard is divided into *halves*, *quarters*, *eighths* and *sixteenths*.

273. The **Surveyor's Chain,** called *Gunter's chain*, used

in measuring roads and boundaries of land, is 4 rods in length, and is divided into 100 links.

Thus, $7\frac{92}{100}$ inches are *1 link;* 100 links, or 4 rods, are *1 chain;* and 80 chains are 1 mile.

274.—1. How many inches are 2 feet? Are 6 feet?

2. How many inches is 1 yard? Are 2 yards?

3. How many feet are 6 yards? Are 8 yards?

4. How many feet in 24 inches? In 72 inches?

5. How many yards in 36 inches? How many feet in 72 inches? How many yards in 72 inches?

6. How many yards in 24 feet? In 36 feet?

7. How many yards in 3 rods? In 5 rods?

8. In 1 mile how many rods? In $\frac{1}{2}$ of a mile how many rods?

SOLUTION.—Since in 1 mile there are 320 rods, in $\frac{1}{2}$ of a mile there must be one half of 320 rods, or 160 rods.

9. How many rods in $\frac{1}{4}$ of a mile? In $\frac{1}{8}$ of a mile?

10. How many inches in 3 quarters of a yard?

11. How many eighths in 3 quarters of a yard?

12. How many yards in 33 feet? In 45 feet?

13. How many rods in 33 feet? In 66 feet?

14. What part of a foot is 4 inches? Is 8 inches?

15. At 5 cents a foot, what will $10\frac{1}{2}$ yards of wire cost?

16. When ribbon is worth 20 cents a yard, what are 3 yards and 3 quarters worth?

17. What part of a yard is 9 inches? Is 45 inches?

18. At $\frac{1}{4}$ of a dollar a rod, how much will it cost to construct a path $\frac{1}{4}$ of a mile long?

DEFINITIONS.

275. **Denomination** is the name of the unit expressing a measure or number.

Of two denominations, the *higher* is that which expresses the greater value, and the *lower* is that which expresses the less value.

276. A **Denominate Number** is a number expressed in one or more denominations.

WRITTEN EXERCISES.

277.—1. How many feet are 46 rods?

$$\begin{array}{r} 46 \text{ rd.} \\ 5\frac{1}{2} \\ \hline 230 \\ 23 \\ \hline 253 \text{ yd.} \\ 3 \\ \hline 759 \text{ ft.} \end{array}$$

SOLUTION.—Since 1 rod is $5\frac{1}{2}$ yards, there must be $5\frac{1}{2}$ times as many yards as rods; hence, in 46 rods there must be $5\frac{1}{2}$ times 46 yards, or 253 yards.

Since 1 yard is 3 feet, there must be 3 times as many feet as yards; hence, in 253 yards, there are 3 times 253 feet, or 759 feet, which is the answer required.

2. How many rods are 759 feet?

$$\begin{array}{rr} 3 \text{ ft.}) & 759 \text{ ft.} \\ 5\frac{1}{2} \text{ yd.}) & 253 \text{ yd.} \\ 2 & 2 \\ \hline 11 \text{ hf. yd.}) & 506 \text{ hf. yd.} \\ & 46 \text{ rd.} \end{array}$$

SOLUTION.—Since 3 feet are one yard, 759 feet must be as many yards as there are times 3 feet in 759 feet, which are 253 times. Hence, 759 feet = 253 yards.

Since $5\frac{1}{2}$ yards are 1 rod, 253 yards must be as many rods as there are times $5\frac{1}{2}$ yards in 253 yards, or times 11 half yards in 506 half yards, which are 46 times. Hence, 253 yards, or 759 feet = 46 rods.

3. How many feet are 80 rods? Are 15 miles?
4. How many rods are 1320 feet? Are 4700 feet?
5. How many miles are 79200 feet? Are 7920 chains?

SURFACE MEASURES.

278. A **Surface** is that which has length, and breadth or width, without thickness.

279. A **Square** is a figure having four equal straight sides, and four equal corners or angles.

280. A **Square Inch** is a square having each of its sides 1 inch in length.

1 Square Inch.

281. A **Square Foot** is a square having each of its sides 1 foot in length.

282. A **Square Rod** is a square having each of its sides 1 rod in length.

283. A **Square Mile** is a square having each of its sides 1 mile in length.

284. **Surface Measures** are those used in ascertaining extent of surface.

The units of surface are a *square inch*, a *square foot*, a *square yard*, a *square rod*, an *acre* and a *square mile*.

TABLE.

144 square inches (sq. in.) are 1 square foot . . . sq. ft.
9 square feet " *1 square yard . sq. yd.*
$30\frac{1}{4}$ square yards " *1 sq. rod or perch. . P.*
160 sq. rods or perches " *1 acre . . . . . A*
640 acres " *1 square mile . . sq. m.*

1 A. = 160 P. = 4840 sq. yd. = 43560 sq. ft. = 6272640 sq. in.

285. In the **Measurement of Land**, 16 square rods are *1 square chain* (*sq. ch.*), and 10 square chains are 1 acre.

286.—1. How many square inches in 2 square feet?

2. How many square feet in 4 square yards? In 8 square yards? In 2 square rods? In 3 square rods?

3. How many square yards in 36 square feet? In 72 square feet?

4. How many square rods in 1 acre? In $\frac{1}{2}$ of an acre?

5. What part of an acre is 80 square rods? Is 40 square rods?

6. How many square rods are 27 square feet?

7. How many square rods are $60\frac{1}{2}$ square yards?

8. What part of an acre is 5 square chains?

9. What will it cost to pave 108 square feet of a walk, at 50 cents a square yard?

WRITTEN EXERCISES.

287.—1. How many square feet in 25 acres?

SOLUTION.

25 A.
160
1500
25
4000 P.
$30\frac{1}{4}$
120000
1000
121000 sq. yd.
9
1089000 sq. ft.

2. How many acres in 1089000 square feet?

SOLUTION.

9 sq. ft.) *1089000 sq. ft.*
$30\frac{1}{4}$ *sq. yd.*) *121000 sq. yd.*
4 *4*
121 fourths sq. yd.) *484000 fourths sq. yd.*
160 P.) *4000 P.*
25 A.

3. How many square inches in 15 square rods?

4. How many square rods in 588060 square inches?

5. In 100 square chains how many square rods?
6. In 1600 square rods how many square chains?
7. In 36 square miles how many square rods?
8. In 23040 square rods how many square miles?
9. In 120 square yards how many square inches?
10. How many acres are there in a lot of land 80 rods long and 72 rods wide?
11. How many acres of land are there in a road $6\frac{1}{2}$ miles long and 5 rods wide?

CUBIC MEASURES.

288. A **Solid,** or **Volume,** is that which has length, breadth and thickness, or depth.

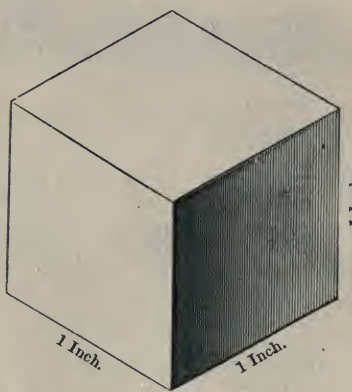
289. A **Cube** is a solid bounded by six equal squares, called faces.

290. A **Cubic Inch** is a cube whose faces are each 1 inch square.

291. A **Cubic Foot** is a cube whose faces are each 1 foot square.

292. A **Cubic Yard** is a cube whose faces are each 1 yard square.

293. Cubic Measures are those used in measuring things that have length, breadth and depth, or thickness.



A Cubic Inch.

The units are a *cubic inch,* a *cubic foot* and a *cubic yard;* also, a *cord foot* and a *cord.*

TABLE.

1728 cubic inches (cu. in.) are 1 cubic foot...cu. ft.
27 cubic feet " 1 cubic yard..cu. yd.

Also,

16 cubic feet are 1 cord foot.....cd. ft.
8 cord feet, or 128 cubic feet, } are 1 cord.....cd.

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

294. **Wood** as usually cut for the market is 4 feet long, and is piled in ranges 4 feet high. Of such ranges, a part that is 1 foot of the length of the range is 1 cord foot, or *1 foot of wood;* and a part that is 8 feet of the length of the range is *1 cord of wood.*

295.—1. How many cubic inches in 2 cubic feet?

2. How many cubic feet in 2 cubic yards?

3. How many cubic feet in 2 cord feet? In 3 cord feet? In 4 cord feet? In $\frac{1}{8}$ of a cord? In $\frac{1}{2}$ of a cord?

4. How many cords in 48 cord feet? In 88 cord feet?

WRITTEN EXERCISES.

296.—1. How many cubic inches in 306 cubic yards?

SOLUTION.

306 cu. yd.
27
2142
612
8262 cu. ft.
1728
66096
16524
57834
8262
14276736 cu. in.

2. How many cubic yards in 14276736 cubic inches?

SOLUTION.

1728)14276736(8262 cu. ft.
13824
4527
3456
10713
10368
3456
3456

27)8262(306 cu. yd.
81
162
162

3. How many cubic feet in 365 cords?

4. How many cords in 46720 cubic feet?

---

297. TEST QUESTIONS.—1. For what is linear or long measure used? What are its units? Recite the table.

2. How is the yard divided in cloth measure? How many inches in 1 hand? How many feet in 1 pace? In 1 fathom? What is the length of Gunter's chain?

3. What is a denomination? Of two denominations, which is the higher? The lower? What is a denominate number?

4. What is a surface? A square? A square inch? A square foot? A square rod? A square mile? Recite the table.

5. What is a volume or solid? A cube? A cubic inch? A cubic foot? A cubic yard? For what are cubic measures used? What are the units of cubic measures? Recite the table. How is wood usually cut and ranged?

SECTION XXIV.

MEASURES OF CAPACITY.

LIQUID MEASURES.

298. Liquid Measures are used in measuring liquids. The units are a *gill*, a *pint*, a *quart* and a *gallon*.

TABLE.

4 gills (gi.) are 1 pint . . . . . pt.
2 pints " 1 quart. . . . qt.
4 quarts " 1 gallon . . . gal.

1 gal. = 4 qt. = 8 pt. = 32 gi.

A *Barrel*, regarded as a measure of cisterns, vats, etc., is $31\frac{1}{2}$ gal., and a *Hogshead* is 63 gallons.

A *Gallon*, liquid measure, contains 231 cubic inches.

299.—1. How many gills in 8 pints? In 12 pints?

2. How many pints in 32 gills? In 48 gills?

3. How many pints in 8 quarts? In 11 quarts?

4. How many quarts in 16 pints? In 24 pints?

5. How many quarts in 6 gallons? In 10 gallons?

6. How many gallons in 24 quarts? In 48 quarts?

7. What part of a quart is 2 gills? Is 6 gills?

8. What part of a gallon is 1 quart? Is 3 quarts?

9. At 40 cents a gallon, what will 3 quarts of wine cost?

10. How many pint-and-a-half bottles can be filled from a gallon and a half?

11. How many barrels can be filled from fifty hogsheads?

12. How many quarts are there in a hogshead of molasses?

WRITTEN EXERCISES.

300.—1. How many gills in 84 gallons?

SOLUTION.

84 gal.
4
336 qt.
2
672 pt.
4
2688 gi.

2. How many gallons in 2688 gills?

SOLUTION.

4)2688 gi.
2)672 pt.
4)336 qt.
84 gal.

3. How many pints in 116 hogsheads?

4. How many hogsheads in 928 pints?

5. At 3 cents a gill, what will a barrel of liquor cost?

6. At 6 cents a quart, how much must be paid for 40 gallons of milk?

DRY MEASURES.

301. **Dry Measures** are those used in measuring grain, fruit, coal, salt, and similar articles.

The units are a *pint*, a *quart*, a *peck* and a *bushel*.

TABLE.

2 pints (pt.) are 1 quart. . . qt.
8 quarts " 1 peck. . . . pk.
4 pecks " 1 bushel. . bu.

$1\ bu. = 4\ pk. = 32\ qt. = 64\ pt.$

A *Gallon*, or 4 quarts, dry measure, contains $268\frac{4}{5}$ cubic inches; and a *bushel*, $2150\frac{42}{100}$ cubic inches.

302.—1. How many pints in 6 quarts? In 12 quarts?

2. How many quarts in 8 pecks? In 11 pecks?

3. How many quarts in 12 pints? In 24 pints?

4. How many pecks in 64 quarts? In 88 quarts?

5. How many pecks in 9 bushels? In 12 bushels?

6. How many bushels in 36 pecks? In 60 pecks?

7. What will $2\frac{1}{2}$ quarts of chestnuts cost, at 5 cents a pint?

8. What will $3\frac{3}{4}$ gallons of milk cost, at 4 cents a quart?

9. What will a peck and a half of berries cost, at $16\frac{2}{3}$ cents, or $\frac{1}{6}$ of a dollar, a quart?

WRITTEN EXERCISES.

303.—1. In 310 bushels, how many quarts?

2. In 9920 quarts, how many bushels?

3. How many pints in 64 pecks?

4. How many bushels in 40960 pints?

5. At 8 cents a quart, how much are 6 bushels of chestnuts worth?

6. At 8 cents a quart, how many bushels of chestnuts can be bought for $15.36?

---

304. TEST QUESTIONS. — 1. What are liquid measures? Name the units of liquid measures. Recite the table.

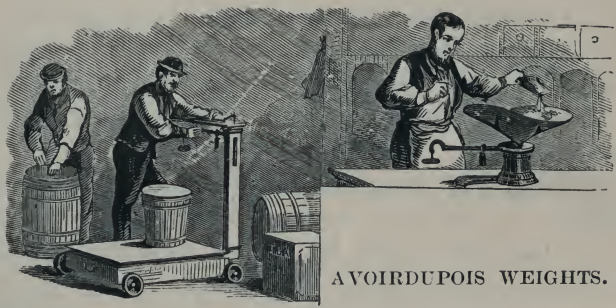
2. How many gallons in a barrel when it is regarded as a measure? A hogshead? How many cubic inches does a gallon contain?

3. What are dry measures? Name the units of dry measures. Recite the table.

4. How many cubic inches does a gallon contain? How many does a bushel contain?

SECTION XXV.

MEASURES OF WEIGHT.



AVOIRDUPOIS WEIGHTS.

305. **Avoirdupois Weights** are those used in weighing produce, groceries, coal, iron, and similar articles.

The units are an *ounce*, a *pound*, a *hundred-weight* and a *ton*.

TABLE.

16 ounces (oz.)	*are*	*1 pound* ..... *lb.*
100 pounds	"	*1 hundred-weight* .. *cwt.*
20 hundred-weight	"	*1 ton* ..... *T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz.

Also, 56 pounds of corn or rye, 60 pounds of wheat or potatoes, or 32 pounds of oats, are *1 bushel;* 100 pounds of dry fish are *1 quintal;* 100 pounds of grain are *1 cental;* and 196 pounds of flour, or 200 pounds of beef or pork, are *1 barrel.*

306. At the **Custom Houses,** in collecting duties on English goods, and in the wholesale and freighting of coal, 28 pounds are *1 quarter*, 4 quarters, or 112 pounds, are 1 hundred-weight, and 20 hundred-weight, or 2240 pounds, are 1 ton, called the *long ton.*

307.—1. How many ounces in 2 pounds? In 3 pounds? In 5 pounds?

2. How many pounds in 32 ounces? In 48 ounces?

3. How many pounds in 6 hundred-weight? In 9 hundred-weight?

4. How many hundred-weight in 500 pounds? In 700 pounds?

5. How many tons in 50 hundred-weight? In 100 hundred-weight?

6. What part of a pound is 12 ounces?

7. What part of a hundred-weight is 25 pounds? Is 75 pounds?

8. How many pounds in 1 quarter of a hundred-weight? In 3 quarters of a hundred-weight?

9. At \$2 a hundred-weight, what will $1\frac{1}{2}$ tons of iron cost?

10. At 10 cents an ounce, what will $1\frac{1}{4}$ pounds of rhubarb cost?

11. At \$1.50 per bushel, how much must be paid for a bag of wheat weighing 90 pounds?

WRITTEN EXERCISES.

308.—1. How many ounces in 316 pounds?

2. How many pounds in 65 tons?

3. How many pounds in 5056 ounces? How many tons in 13000 pounds?

4. How many bushels in a load of wheat weighing 4590 pounds?

5. What is the weight of $76\frac{1}{2}$ bushels of wheat?

6. At $1\frac{1}{2}$ cents per pound, what will be the cost of freighting 16 tons of goods?

TROY WEIGHTS.

309. Troy Weights are those used in weighing gold, silver and gems.

The units are a *grain*, a *pennyweight*, an *ounce* and a *pound*.

TABLE.

24 grains (gr.) are 1 pennyweight....pwt.
20 pennyweights " 1 ounce.....oz.
12 ounces " 1 pound.....lb.

1 lb. = 12 oz. = 240 pwt. = 5760 gr.

A *Pound Avoirdupois* is equal to 7000 Troy grains.

310. Apothecaries, in compounding medicines and in putting up prescriptions, either use only the denominations of grains, ounces and pounds, or subdivide the Troy pound into *grains, scruples, drams* and *ounces.* Thus,

20 grains (gr.) are 1 scruple....℈.
3 scruples " 1 dram.....ʒ.
8 drams " 1 ounce.....℥.
12 ounces " 1 pound..... lb.

311.—1. How many grains in 1 pennyweight? In 2 pennyweights?

2. How many pennyweights in 48 grains?

3. How many pennyweights in 2 ounces? In 5 ounces?

4. How many ounces in 40 pennyweights? In 100 pennyweights?

5. How many ounces in 3 pounds? In 10 pounds?

6. How many pounds in 48 ounces? In 144 ounces?

7. At $3.25 a pennyweight, what is the value of a jewel weighing 12 pennyweights?

8. At 3 cents a scruple, what is the value of a drug weighing 1 ounce?

9. How many doses, of 5 grains each, are there in 1 dram of medicine?

WRITTEN EXERCISES.

312.—1. How many grains in 65 ounces?

2. How many ounces in 31200 grains?

3. How many pennyweights in $51\frac{1}{2}$ pounds?

4. How many pounds in 12360 pennyweights?

5. At 25 cents a dram, what will 80 pounds of drugs cost?

6. At 25 cents a dram, how many pounds of drugs may be bought for $1920?

7. If the gold coin of the United States is composed of 9 parts of pure gold and 1 part of alloy, how many pennyweights of alloy are there in 20 pounds of coin?

---

313. TEST QUESTIONS.—1. What are avoirdupois weights? Name the units of avoirdupois weights. Recite the table.

2. How many pounds of corn or rye are 1 bushel? How many pounds of wheat or potatoes? How many of oats?

3. How many pounds of fish are 1 quintal? How many

pounds of grain are 1 cental? How many pounds of flour are 1 barrel? How many pounds of beef or pork are 1 barrel?

4. How many pounds are a quarter in the wholesale and freighting of coal? How many pounds are a hundred-weight? How many pounds in a ton?

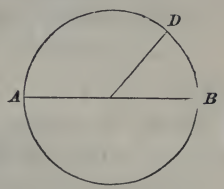
5. What is Troy weight? What are the units of Troy weight? Recite the table. Recite the table of apothecaries' weights.

6. How many Troy grains are 1 pound Troy? How many Troy grains are equal to 1 pound avoirdupois? Which is the heavier, a pound Troy or a pound avoirdupois?

SECTION XXVI.

CIRCULAR MEASURES.

314. A **Circle** is a plane surface bounded by a line, all parts of which are equally distant from a point within called the center.

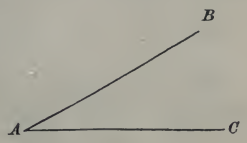


315. A **Circumference** is the line that bounds a circle.

316. An **Arc** is any part of the circumference; as AD, or DB.

317. A **Degree** is one of the 360 equal parts of a circumference.

318. An **Angle** is the difference of direction of two lines which meet at a point.

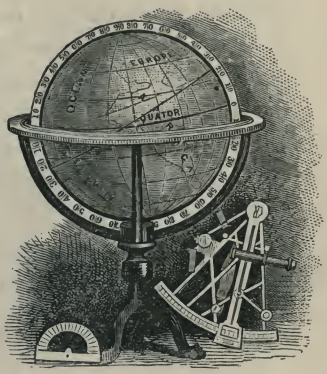


Thus, the lines AB and AC, which meet at A, form the angle BAC.

319. The **Measure of an angle** whose sides meet at

the center of a circle is that part of the circumference between the sides.

Thus, the arc *AD* is the measure of the angle *ACD* which is formed in the circle, (page 146).



320. Circular Measures are those used in measuring the arcs of circles, angles and the difference of directions.

The units are a *second*, a *minute*, a *degree* and a *circumference*.

TABLE.

60 seconds (″) are 1 minute . . . . . ′.
60 minutes " 1 degree . . . . . °.
360 degrees " 1 circumference . . C.

$$1\ C. = 360° = 21600' = 1296000''.$$

321. A **Minute** of the circumference of the earth, or a geographic mile, is about $1\frac{1}{6}$ common miles.

322.—1. How many seconds in 2 minutes? In 3 minutes? In 5 minutes?

2. How many minutes in 120 seconds? In 180 seconds? In 300 seconds?

3. How many minutes in 3 degrees? In 4 degrees?

4. How many degrees in 120 minutes? In 240 minutes? In 360 minutes?

5. What part of a circumference is 30 degrees?

6. How many degrees in $\frac{1}{4}$ of a circumference?

WRITTEN EXERCISES.

323.—1. How many seconds in 240 degrees?

2. How many degrees in 864000 seconds?

3. How many minutes in $\frac{3}{4}$ of a circumference?

4. How many degrees in 14200 minutes?

5. How many seconds in $\frac{2}{3}$ of an hour?

6. If a vessel sail $2\frac{1}{2}$ degrees of the circumference of the earth in one day, in how many days will it sail 180 degrees?

7. In sailing 180 degrees of the circumference of the earth, how many common miles does a vessel pass over?

SECTION XXVII.

MEASURES OF TIME.

324. Measures of Time are those used in measuring time or duration.

The units are a *second*, a *minute*, an *hour*, a *day* and a *year*.

TABLE.

60 seconds (s.)	*are*	*1 minute .....m.*
60 minutes	"	*1 hour .....h.*
24 hours	"	*1 day .....d.*
365 days	"	*1 common year .c. y.*
366 days	"	*1 leap year .....l. y.*

1 c. y. = 365 d. = 8760 h. = 525600 m. = 31536000 s.

Also, 7 days are *1 week*, 12 months are *1 year*, and 100 years are *1 century*.

325. The *Names of the Months*, and the number of days in each, are—

	Days.		Days.
January, 1st month,	31.	July, 7th month,	31.
February, 2d "	28 or 29.	August, 8th "	31.
March, 3d "	31.	September, 9th "	30.
April, 4th "	30.	October, 10th "	31.
May, 5th "	31.	November, 11th "	30.
June, 6th "	30.	December, 12th "	31.

The exact length of the year is about 365¼ days; hence, every fourth year February has 29 days, and the year has 366 days.

In general, every year whose number can be divided by 4 without a remainder is leap year.

Thus, the year 1872 is a leap year.

326.—1. How many seconds in 1 minute? In one half a minute? In one third of a minute?

2. How many minutes in 120 seconds?

3. How many minutes in 1 hour? In ½ of an hour?

4. How many hours in 120 minutes? In 180 minutes? In 240 minutes?

5. How many hours in 2 days? In 3 days?

6. What part of a day is 6 hours? Is 18 hours?

7. How many days in 48 hours? What part of a day are 6 hours?

8. How many months in 5 years? In 9 years?

9. How many weeks in 49 days? In 63 days?

10. How many days has a leap year? A common year?

11. What months have 30 days each? In what years does February have 29 days?

12. How many days are there in 7 weeks? In 9 weeks?

WRITTEN EXERCISES.

327.—1. How many minutes in 18 days?

2. How many days in 25920 minutes?

3. How many hours in a common year?

4. How many seconds in 24 hours?

5. How many hours in 86400 seconds?

6. How many years in 8760 hours?

7. How many seconds in a common year?

8. How many days in 31622400 seconds?

9. If you are 11 years old, how many minutes have you lived, allowing $365\frac{1}{4}$ days to a year?

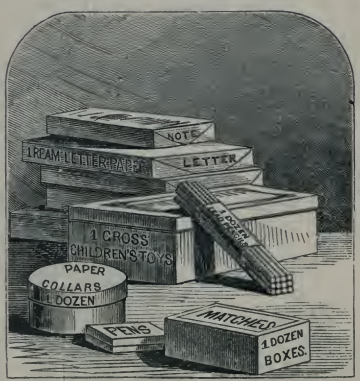
SECTION XXVIII.

PAPER AND COUNTING.

328. Paper is bought and sold by the *sheet*, *quire*, *ream*, *bundle* or *bale*.

TABLE.

24 sheets are 1 quire.
20 quires " 1 ream.
2 reams " 1 bundle.
5 bundles " 1 bale.



329. In counting certain articles, the units *dozen*, *gross* and *great gross* are used.

TABLE.

12 things are 1 dozen . . . . . doz.
12 dozen " 1 gross . . . . . gro.
12 gross " 1 great gross . . grt. gro.

Two things are a *pair*, and 20 things are a *score*.

330.—1. How many sheets of paper in 2 quires?

2. How many quires in 1 bundle? In 2 bundles?

3. How many quires in 48 sheets?

4. How many reams in 80 sheets? In 100 sheets?

5. How many things in 5 dozen? How many pens in a gross?

6. How many dozen in a great gross? In half of a great gross?

7. How many pairs are 40? How many scores are 40? Are 80? Are 120?

WRITTEN EXERCISES.

331.—1. How many sheets are $9\frac{1}{2}$ reams?

2. How many reams are 4560 sheets?

3. How much will a gross of pens cost, at 16 cents a dozen?

4. If you buy $5\frac{3}{10}$ reams of paper, how many sheets will you obtain?

5. In some boxes there are 2064 eggs; what is their value at 30 cents a dozen?

6. How many boxes will be required to pack 2592 screws, if each box will hold 9 dozen?

7. How much will 25 gross of pens cost, at 2 cents per pen?

8. What will 75 bundles of paper cost, at 15 cents per quire?

332. TEST QUESTIONS.—1. What is a circle? A circumference? An arc? A degree?

2. What is an angle? The measure of an angle?

3. What are circular measures? The units of circular measures? Recite the table.

4. How many miles is a minute of circumference?

5. What are measures of time? The units of measures of time? Recite the table.

6. How many days are 1 week? How many months are 1 year? How many years are 1 century?

7. Name the months, and give the number of days in each.

8. What is the exact length of a year? How often has February 29 days? When February has 29 days, how many days has the year? In general, what years are leap years?

9. By what is paper bought and sold? Recite the table. What units are used in counting certain articles? Recite the table. How many things are a pair? How many things are a score?

SECTION XXIX.

REVIEW OF DENOMINATE NUMBERS.

333.—1. How many inches in $\frac{2}{3}$ of a yard?

SOLUTION.—1 yard is 3 feet; $\frac{2}{3}$ of 3 feet are 2 feet; and since 1 foot is 12 inches, 2 feet are 2 times 12 inches, or 24 inches. Hence, $\frac{2}{3}$ of a yard are 24 inches.

2. How many pints in $\frac{3}{4}$ of a bushel?

3. At $\frac{3}{4}$ of a dollar a cord foot, what will $\frac{7}{8}$ of a cord of wood cost?

4. At \$.10 per ounce, what will $1\frac{1}{2}$ pounds of spice cost?

5. What part of a bushel is 3 pecks?

6. At 2 dollars a bushel, what will 3 pecks of wheat cost?

7. James is 108 months old, and his age is $\frac{3}{4}$ of his brother's. What is his brother's age?

8. At 16 cents a yard, what will 9 yards of cloth cost?

9. At 12 cents a peck, how many bushels of oats can be bought for 96 cents?

10. If 40 perches of land cost 9 dollars, what will an acre cost?

11. If $\frac{5}{10}$ of a ton of hay be worth 3 dollars, what will 30 hundred-weight be worth?

WRITTEN EXERCISES.

334.—1. How many minutes in a leap year?

2. A walk is 40 rods long; what will it cost to pave it, at 25 cents per linear foot?

3. Into how many lots, of 20 square rods each, can an acre and a half be divided?

4. How much must be paid for 25 barrels of beef, at 11 cents a pound?

5. At 11 cents a pound, how many pounds of beef can be bought for $550?

6. For how much will a hogshead of liquor sell, if retailed at 10 cents a half gill?

7. A certain town is 36 square miles in extent; how many acres does it contain?

8. How many bushels in a load of corn, which, at 86 cents a bushel, costs $34.40?

9. How many hogsheads of molasses, of 63 gallons each, worth 55 cents a gallon, can be bought for $1386?

SECTION XXX.

REDUCTION OF COMPOUND NUMBERS.

335.—1. How many inches are 6 feet 5 inches?

SOLUTION.—Since 1 foot is 12 inches, 6 feet must be 6 times 12 inches, or 72 inches; and 72 inches plus 5 inches are 77 inches. Hence, 6 feet 5 inches are 77 inches.

2. How many cord feet in 5 cords 6 cord feet?

3. Reduce 1 peck 3 quarts 1 pint to pints.

4. Reduce 2 tons 9 hundred-weight to hundred-weights.

5. How many feet in 77 inches?

SOLUTION.—Since 12 inches are 1 foot, 77 inches must be as many feet as 12 inches are contained times in 77 inches, which are 6 times, with a remainder of 5 inches. Hence, 77 inches are 6 feet 5 inches.

6. How many cords in 46 cord feet?

7. Reduce 37 gills to quarts.

8. How many pecks in 33 pints?

9. How many tons are 49 hundred-weight?

10. What will 2 gallons 3 quarts of vinegar cost, at 5 cents a pint?

11. A wall is 4 yards 6 inches long; what is its length in inches?

DEFINITIONS.

336. A **Simple Denominate Number** is a number expressed in units of only one denomination.

Thus, 5 yards is a simple denominate number.

337. A **Compound Denominate Number** is a number expressed in units of more than one denomination.

Thus, 6 yards 7 inches is a compound denominate number.

338. Reduction of Denominate Numbers is the process of changing them from one denomination to another without changing their value.

339. Principle.—*Denominate numbers are reduced to lower denominations by multiplication, and to higher denominations by division.*

REDUCTION DESCENDING.

340. Reduction Descending is the process of changing a number to an equivalent number expressed in units of a lower denomination.

WRITTEN EXERCISES.

341.—1. How many pints are 3gal. 2qt. 1pt.?

3 gal. 2 qt. 1 pt.
4
14 qt.
2
29 pt.

SOLUTION.—Since 1 gallon is 4 quarts, 3 gallons must be 3 times 4 quarts, or 12 quarts; and 12 quarts plus 2 quarts are 14 quarts.

Since 1 quart is 2 pints, 14 quarts must be 14 times 2 pints, or 28 pints; and 28 pints plus 1 pint are 29 pints.

Hence, 3gal. 2qt. 1pt. are 29pt.

2. Reduce 4yd. 2ft. 7in. to inches.

3. Reduce 5 sq. yd. 7 sq. ft. 27 sq. in. to square inches.

342. Rule for Reduction Descending.—*Multiply the number of the highest denomination given, by that number of the next lower which equals one of the higher, and to the product add the number, if any, of the next lower denomination.*

Reduce this result in like manner, and so proceed until the given number is reduced to the required denomination.

PROBLEMS.

343.—1. Reduce 3° 4′ 16″ to seconds.

2. Reduce 3A. 140P. to square yards.

3. Reduce 1mi. 138rd. 4yd. to feet.

4. Reduce 100 cu. yd. 20 cu. ft. 100 cu. in. to cubic inches.

5. Reduce 7bu. 3pk. 5qt. to quarts.

6. Reduce 9oz. 11pwt. 17gr. to grains.

7. Reduce 4T. 9cwt. 9lb. to ounces.

8. Reduce 1hhd. 20gal. 3qt. to quarts.

9. How much must be paid for 1A. 40 sq. rd. of land, at $1.50 per square rod?

10. What will it cost to make 1mi. 100rd. of fence, at $1.20 a rod?

11. How much must be paid for 1℥ 6℥ 2℈ of medicine, at 5 cents a grain?

12. How many hours has a man lived who is 50y. 125d. old, allowing $365\frac{1}{4}$ days to the year?

REDUCTION ASCENDING.

344. **Reduction Ascending** is the process of changing a number to an equivalent number, expressed in units of a higher denomination.

WRITTEN EXERCISES.

345.—1. How many gallons are 29 pints?

2 pt.)29 pt.
4 qt.)14 qt. 1 pt.
3 gal. 2 qt.

29 pt.=3 gal. 2 qt. 1 pt.

SOLUTION.—Since 2 pints are 1 quart, 29 pints must be as many quarts as 2 pints are contained times in 29 pints, which are 14 times, with a remainder of 1 pint. Hence, 29 pints are 14 quarts 1 pint.

Since 4 quarts are 1 gallon, 14 quarts must be as many gallons as 4 quarts are contained times in 14 quarts, which are 3 times, with a remainder of 2 quarts. Hence, 14 quarts are 3 gallons 2 quarts; and 29 pints are 3 gallons 2 quarts 1 pint.

2. Reduce 175 inches to yards.
3. Reduce 7515 square inches to square yards.
4. Reduce 37038 seconds to hours.

346. **Rule for Reduction Ascending.**—*Divide the given number by that number of its denomination, which equals one of the next higher, and write the remainder, if any.*

Divide the quotient in like manner, and so continue until the given number is changed to the required denomination.

The last quotient, with the remainders, if any, written in their order from the highest to the lowest, will be the required number.

Reductions Descending and Ascending, being performed by opposite processes, are proofs of each other.

PROBLEMS.

347.—1. Reduce 11056 seconds to degrees.
2. Reduce 18755 square yards to acres.
3. Reduce 7569 feet to miles.
4. Reduce 4700260 cubic inches to cubic yards.
5. Reduce 275 quarts to bushels.
6. Reduce 4601 grains to ounces.
7. Reduce 142494 ounces to tons.
8. Reduce 335 quarts to hogsheads.
9. How many acres of land will $300 purchase, at $1.50 per square rod?

10. How many miles of fence, at $1.20 per rod, can be built for $504?

11. How many ounces of medicine, at 5 cents a grain, can be bought for $44?

12. How many years has a man lived who is 438300 hours old, allowing $365\frac{1}{4}$ days to a year?

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348. TEST QUESTIONS.—1. What is a denominate number? A simple denominate number? A compound denominate number?

2. What is reduction? The principle of reduction?

3. What is reduction descending? How is a number reduced from a higher to a lower denomination?

4. What is reduction ascending? How is a number reduced from a lower to a higher denomination?

5. Why are reductions descending and ascending proofs of each other?

SECTION XXXI.

ADDITION OF COMPOUND NUMBERS.

349.—1. What is the sum of 6 yards 1 foot, and 4 yards 1 foot?

2. Add 6 cords 3 cord feet, and 5 cords 6 cord feet.

SOLUTION.—6 cords 3 cord feet, and 6 cord feet, are 6 cords 9 cord feet, or 7 cords 1 cord foot; 7 cords 1 cord foot, and 5 cords, are 12 cords and 1 cord foot.

3. Add 10 bushels 2 pecks, and 4 bushels 3 pecks.

4. If you should buy in one month, 4 gallons 3 quarts of kerosene, and the next month, 3 gallons 3 quarts, how much would you buy in all?

5. Mary is 9 years 7 months old, and Alice is 8 years 8 months old. What is the sum of their ages?

6. A farmer sold two turkeys; one weighed 10 pounds 11 ounces, and the other 11 pounds 7 ounces. What was the weight of both?

DEFINITION.

350. Addition of Compound Numbers is the process of uniting two or more compound numbers of the same kind to find their sum.

WRITTEN EXERCISES.

351.—1. What is the sum of 15lb. 5oz. 15pwt.; 7lb. 9oz. 12pwt.; and 6oz. 4pwt.?

15 lb.	*5 oz.*	*15 pwt.*
7	*9*	*12*
	6	*4*
23 lb.	*9 oz.*	*11 pwt.*

SOLUTION.—Write the numbers so that figures expressing units of the same denomination shall be in the same columns.

Begin with the pennyweights, and add the pennyweights, ounces and pounds separately.

The sum of the pennyweights is 31 pennyweights, or 1 ounce 11 pennyweights. Write the 11 pennyweights under the column of pennyweights, and reserve the 1 ounce to add with the ounces.

The sum of the ounces is 21 ounces, or 1 pound 9 ounces. Write the 9 ounces under the column of ounces, and reserve the 1 pound to add with the pounds.

The sum of the pounds is 23 pounds, which write under the column of pounds.

The entire sum is 23 pounds 9 ounces 11 pennyweights.

2. What is the sum of 6mi. 120rd. 3yd.; 7mi. 160rd. 5yd.; and 55rd. 3yd.?

3. What is the sum of 15gal. 3qt. 1pt.; 7gal. 0 qt. 1pt.; and 2qt. 0pt.?

352. **Rule for Addition of Compound Numbers.**— *Write the numbers so that units of the same denomination shall stand in the same column.*

Begin with the lowest denomination, and add the numbers of each denomination separately. If the sum is less than one of the next higher denomination, write it as a part of the required result.

If the sum is equal to or exceeds one of the next higher denomination, reduce it to that denomination, write the remainder, if any, as a part of the required result, and add the units of the higher denomination with the column of that denomination.

PROBLEMS.

353. Write and add—

(1.)

cwt.	*lb.*	*oz.*
15	75	7
3	0	5
	25	9

(2.)

A.	*sq. rd.*
10	146
73	15
3	75

(3.)

cu. yd.	*cu. ft.*	*cu. in.*
5	17	703
11	10	835
	11	106
3	9	112

(4.)

oz.	*pwt.*	*gr.*
9	18	11
1	19	23
4	0	20
6	0	0

(5.)

bu.	*pk.*
100	3
76	2
13	3
	3

(6.)

rd.	*yd.*	*ft.*
25	3	2
16	2	1
11	3	0
	4	2

7. What is the sum of 2d. 7h. 6m.; 5d. 13h. 25m.; 11h. 11m.; and 7d. 15h. 55m.?

8. Add 1° 30′ 15″ 19° 45′ 17″; and 31° 40′ 16″.

9. A boy gathered in one day, 1pk. 3qt. 1pt. of berries; another day, 1pk. 5qt. 1pt.; and a third day, 2pk. 2qt. 1pt. How many berries did he gather in all?

10. In one field there are 17A. 120 sq. rd.; and in another 15A. 140 sq. rd. How much land is there in both fields?

11. In one car there are 16T. 11cwt. 75lb. of coal; in another, 15T. 19cwt. 31lb.; and in a third, 14T. 17cwt. 50lb. How much coal is there in the three cars?

SECTION XXXII.

SUBTRACTION OF COMPOUND NUMBERS.

354.—1. In one sack there are 2bu. 3pk. of wheat, and in another, 1bu. 2pk. How much more is there in one sack than in the other?

2. If 5ft. 10in. be cut from a line 7ft. 9in. long, how much of the line will be left?

SOLUTION.—There will be left of the line the difference between 7 feet 9 inches and 5 feet 10 inches. 7 feet 9 inches less 10 inches are 6 feet 11 inches; and 6 feet 11 inches less 5 feet are 1 foot 11 inches. Hence, there will be left 1 foot 11 inches.

3. If you should have in a cask 11gal. 3qt. of molasses, and should sell 8gal. 3qt. of it, how much would there be left?

4. From 10h. 30m. subtract 8h. 40m.

5. In one firkin there are 12lb. 8oz. of butter, and in another 9lb. 12oz. How much more is there in the one than in the other?

6. In one load there are 2cd. 1 cd. ft. of wood, and in another load 1cd. 7 cd. ft. How much is required to make the smaller load equal the greater?

DEFINITION.

355. Subtraction of Compound Numbers is the process of finding the difference between two compound numbers of the same kind.

WRITTEN EXERCISES.

356.—1. From 17bu. 2pk. 6qt. take 8bu. 3pk. 4qt.

17 bu.	*2 pk.*	*6 qt.*
8	*3*	*4*
8 bu.	*3 pk.*	*2 qt.*

SOLUTION. — Write the subtrahend under the minuend, so that units of the same kind shall stand in the same columns.

Begin at the right and subtract the units of each denomination of the subtrahend from those of the same denomination in the minuend.

4 quarts from 6 quarts leaves 2 quarts, which is the difference of the quarts.

Since 3 pecks cannot be taken from 2 pecks, take 1 bushel from the 17 bushels, leaving 16 bushels, and add it, reduced, to the 2 pecks, thus obtaining 6 pecks; then, 3 pecks from 6 pecks leaves 3 pecks, which is the difference of the pecks.

8 bushels from 16 bushels leaves 8 bushels, which is the difference of the bushels.

The entire difference is 8 bushels 3 pecks 2 quarts.

2. From 13lb. 5oz. 16pwt. subtract 9lb. 7oz. 14pwt.

3. From 6gal. 2qt. 0pt. subtract 4gal. 3qt. 1pt.

4. From 22yd. 2qr. 2na. subtract 13yd. 3qr. 3na.

357. Rule for Subtraction of Compound Numbers. — *Write the subtrahend under the minuend, so that units of the same denomination shall be in the same column.*

Begin with the lowest denomination, and subtract the units of each denomination of the subtrahend from those of the same denomination in the minuend, if possible, and write the difference beneath.

If the number of any denomination of the subtrahend is greater than that of the same denomination in the minuend, increase the number in the minuend, by adding to it as many units as make one of the next higher denomination, and subtract; then, regarding the number of the next higher denomination of the minuend as one less, proceed as before.

PROBLEMS.

358. Write and subtract—

(1.)

From	*25 mi.*	*100 rd.*	*7 yd.*
Take	*16*	*120*	*6*

(2.)

	9 A.	*120 P.*	*24 sq. yd.*
	6	*125*	*19*

(3.)

From	*15 wk.*	*0 d.*	*10 h.*
Take	*12*	*9*	*11*

(4.)

	44 T.	*75 lb.*	*10 oz.*
	21	*80*	*15*

(5.)

From	*11* ♃	*0* ♃	*14* ♄
Take	*7*	*2*	*16*

(6.)

	3°	*7′*	*0″*
	1	*15*	*45*

7. A man who has started on a journey of 63mi. 160rd. has traveled 44mi. 125rd. 3yd. How much farther has he to travel to complete his journey?

8. James is 13 years 9 months old, and Henry is 11 years 11 months old. What is the difference in their ages?

9. From a cask which contained 75 gallons of vinegar there has been drawn 46gal. 3qt. 1pt. How much remains in the cask?

10. The longitude of Boston is 71° 3′ 30″ West, and of Chicago, 87° 37′ 47″ West. What is the difference of their longitudes?

11. What is the time from May 17, 1869, to June 16, 1871?

1871 y.	*6 mo.*	*16 d.*
1869	*5*	*17*
2 y.	*0 mo.*	*30 d.*

SOLUTION.—The time from May 17, 1869, to May 17, 1871, is 2y.; from May 17, 1871, to June 16, 1871, is 30d. The entire difference of time is 2y. 30d.

12. A man was born May 16, 1819; how old was he July 4, 1871?

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359. TEST QUESTIONS.—1. What is addition of compound numbers? How do you write the numbers for adding?

2. How do you add the units of the several denominations? How do you proceed if the sum of the units of any denomination is less than a unit of the next higher denomination? How do you proceed if their sum is equal to or greater than a unit of the next higher denomination?

3. What is subtraction of compound numbers? How do you write the numbers for subtracting?

4. How do you subtract? How do you proceed if the number of any denomination of the subtrahend is greater than that above it?

SECTION XXXIII.

MULTIPLICATION OF COMPOUND NUMBERS.

360.—1. If a box contain 2 hundred-weight 20 pounds of sugar, how much will 2 similar boxes contain?

2. How much wood is there in 3 ranges, each containing 3 cords 5 cord feet?

SOLUTION.—3 times 5 cord feet are 15 cord feet, or 1 cord 7 cord feet, and 3 times 3 cords are 9 cords; 9 cords and 1 cord 7 cord feet are 10 cords 7 cord feet, the answer required.

3. John is 9 years 4 months old, and his father is 4 times as old. How old is his father?

4. A boy gathered 1 peck 3 quarts of berries every day for 6 days. How many did he gather in all?

5. If it take 1 ounce 3 drams of medicine for a prescription, how much will it take for 6 similar prescriptions?

6. If 4 yards 3 quarters of cloth are required for a suit of clothes, how many yards will be required for 7 suits?

7. How much wood is there in two ranges, each containing 2 cords 6 cord feet?

8. If a team can plow 1 acre 40 square rods in 1 day, how much can it plow in 4 days?

DEFINITIONS.

361. **Compound Multiplication** is the process of taking a compound number as many times as there are units in the multiplier.

WRITTEN EXERCISES.

362.—1. Multiply 6gal. 3qt. 1pt. by 5.

6 gal. 3 qt. 1 pt.
5
34 gal. 1 qt. 1 pt.

SOLUTION.—Write the multiplier under the lowest denomination of the multiplicand.

Begin at the right and multiply the number of each denomination in the order of the denominations.

5 times 1 pint are 5 pints, or 2 quarts 1 pint; write the 1 pint as the number of that denomination in the product, and reserve the 2 quarts to be added to the product of the quarts.

5 times 3 quarts are 15 quarts; 15 quarts and the 2 quarts reserved are 17 quarts, or 4 gallons 1 quart. Write the 1 quart as the number of that denomination in the product, and reserve the 4 gallons to be added to the product of the gallons.

5 times 6 gallons are 30 gallons; 30 gallons and the 4 gallons reserved are 34 gallons, which write as the gallons of the product.

The entire product is 34 gallons 1 quart 1 pint.

2. Multiply 6rd. 2yd. 1ft. by 5.

3. Multiply 10lb. 3pwt. 5gr. by 8.

363. Rule for Multiplication of Compound Numbers.—*Write the multiplier under the lowest denomination of the multiplicand.*

Begin with the lowest denomination, and multiply the number of each denomination in its order.

If the product is less than one of the next higher denomination, write it as a part of the required product.

If the product is equal to or exceeds one of the next higher denomination, reduce it to that denomination, write the remainder, if any, as a part of the required product, and add the units of the higher denomination to the product of that denomination.

PROBLEMS.

364. Copy and multiply—

(1.) *1 hhd. 10 gal. 3 qt.* × *7*

(2.) *50 cu. ft. 1121 cu. in.* × *8*

(3.) *14 A. 63 P. 1 sq. yd.* × *4*

(4.) *4 bu. 3 pk. 2 qt.* × *9*

(5.) *6 wk. 3 d. 10 h.* × *10*

(6.) *13° 0′ 14″* × *8*

7. A farmer sold 6 loads of wood, each containing 2cd. 5 cd. ft. How much did he sell in all?

8. What is the weight of 7 boxes of sugar, if each weighs 1cwt. 31lb. 8oz.?

9. Jane is 8y. 3mo. old; how old is her aunt, who is 5 times as old?

10. If a train of cars move 27mi. 120rd. 2yd. in an hour, how far will it move at the same rate in 11 hours?

11. Flint has in his farm 40A. 35P., and my farm is 7 times as large. What is the extent of my farm?

12. How much molasses is there in 12 casks, each containing 62gal. 3qt. 1pt.?

13. If it require 1oz. 10pwt. 15gr. of silver to make 1 spoon, how much will be required to make 18 spoons?

14. In a certain town there are 3 farms. Each farm is divided into 9 equal lots, and each lot contains 12A. 72P. What is the extent of the 3 farms?

SECTION XXXIV.

DIVISION OF COMPOUND NUMBERS.

365.—1. If 6 pounds 12 ounces of tea be divided equally among 3 persons, how much will each receive?

2. If 8 bushels 3 pecks of chestnuts be divided equally among 5 boys, how many will each boy receive?

SOLUTION. — Each boy will receive one fifth of 8 bushels 3 pecks. One fifth of 8 bushels is 1 bushel, with a remainder of 3 bushels; 3 bushels are 12 pecks, and 12 pecks plus 3 pecks are 15 pecks; one fifth of 15 pecks is 3 pecks. Hence, each boy will receive 1 bushel 3 pecks.

3. Johnson is 37 years 4 months old, and his age is 4 times that of his son. What is the age of his son?

4. If it require 33 yards 1 quarter to make 7 suits, how much will it require to make 1 suit?

5. In 3 equal ranges of wood there are 10 cords 7 cord feet. How much is there in each range?

DEFINITION.

366. Division of Compound Numbers is the process of finding how many times one denominate number is contained in another of a similar kind; or, it is the process of separating a compound number into equal parts.

WRITTEN EXERCISES.

367.—1. Divide 34gal. 1qt. 1pt. by 5.

5)34 gal. 1 qt. 1 pt.
6 gal. 3 qt. 1 pt.

SOLUTION.—Write the divisor at the left of the dividend.

Begin with the highest denomination, and divide the number of each denomination in its order.

One fifth of 34 gallons is 6 gallons, with a remainder of 4

gallons. Write the 6 gallons as the gallons of the quotient; the 4 gallons are 16 quarts, which, added to the 1 quart, give 17 quarts.

One fifth of 17 quarts is 3 quarts, with a remainder of 2 quarts. Write the 3 quarts as the quarts of the quotient; the 2 quarts are 4 pints, which, added to the 1 pint, give 5 pints.

One fifth of 5 pints is 1 pint, which write as a part of the quotient.

The entire quotient is 6 gallons 3 quarts 1 pint.

2. Divide 32rd. 0yd. 1ft. by 5.

3. Divide 81lb. 5pwt. 16gr. by 8.

368. Rule for Division of Compound Numbers.—*Begin with the highest denomination; divide the number of each denomination in its order, and write the several quotients as the parts of the same denominations of the required quotient.*

When there is a remainder reduce it to the next lower denomination, and add the result to the number of that denomination before dividing.

When divisor and dividend are both compound numbers, they must be reduced to simple denominate numbers of the same denomination preparatory to dividing.

Multiplication of compound numbers and division of compound numbers, being performed by opposite processes, are proofs of each other.

PROBLEMS.

369. Copy and divide—

(1.) $7\overline{)7 \text{ hhd. } 45 \text{ gal. } 1 \text{ qt.}}$

(2.) $8\overline{)405 \text{ cu. ft. } 328 \text{ cu. in.}}$

(3.) $4\overline{)14 \text{ A. } 63 \text{ P. } 1 \text{ sq. yd.}}$

(4.) $9\overline{)37 \text{ bu. } 1 \text{ pk. } 2 \text{ qt.}}$

5. If 6 persons should share equally 19cd. 6 cd. ft. of wood, how much would each person receive?

6. Mary's mother is 40y. 3mo. old, and Mary is one fifth as old. How old is Mary?

7. If 7 equal boxes of sugar weigh 7cwt. 27lb. 8oz., what is the weight of one box?

8. If a train of cars move, at a uniform rate, 308m. 44rd. in 11 hours, how far does it move in 1 hour?

9. My farm contains 281A. 85P., and Flint's farm is one seventh as large. How large is Flint's farm?

10. When 22oz. 19pwt. 10gr. of silver are required to make 18 spoons, what is the weight of 1 spoon?

11. A farmer has 65bu. 2pk. 4qt. of grain, which he wishes to put in bags, each containing 1bu. 3pk. 4qt. How many bags will be required?

12. In a certain town there are 3 farms, and each is divided into 9 equal lots. The entire extent of the farms is 37A. 56P.; what is the size of each lot?

13. In what time can a man saw 1 cord of wood, if he can saw 12 cords in 93h. 10min.?

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370. TEST QUESTIONS.—1. What is multiplication of compound numbers? How do you write the numbers for multiplying? How do you multiply?

2. How do you proceed if any product is less than one of the next higher denomination? If any product is equal to one or more units of the next higher denomination?

3. What is division of compound numbers? How do you divide? How do you proceed if there is a partial remainder?

4. Why are the multiplication of compound numbers and the division of compound numbers proofs of each other?

SECTION XXXV.

REVIEW OF COMPOUND NUMBERS.

371.—1. When oats are 10 cents a peck, what will 1 bushel 2 pecks 4 quarts of oats cost?

2. Reduce $\frac{5}{6}$ of a yard to units of lower denominations.

SOLUTION.—Since 1 yard is 3 feet, $\frac{5}{6}$ of a yard is $\frac{5}{6}$ of 3 feet, which is $\frac{15}{6}$ of a foot, or 2 feet and $\frac{3}{6}$, or $\frac{1}{2}$, of a foot. Since 1 foot is 12 inches, $\frac{1}{2}$ of a foot, is $\frac{1}{2}$ of 12 inches, or 6 inches. Hence, $\frac{5}{6}$ of a yard, reduced to units of lower denominations, is 2 feet 6 inches.

3. Reduce $\frac{3}{8}$ of an acre to units of lower denominations.

4. Reduce $\frac{5}{8}$ of a ton to units of lower denominations.

5. 2 gallons 3 quarts are how many pints?

6. Reduce 31 pints to units of higher denominations.

SOLUTION.—31 pints are 15 quarts 1 pint, and 15 quarts are 3 gallons 3 quarts. Hence, 31 pints in units of higher denominations are 3 gallons 3 quarts 1 pint.

7. Reduce 39 inches to units of higher denominations.

8. Reduce 46 ounces Troy to units of higher denominations.

9. How many hours from 9 o'clock A. M. to 2 o'clock P. M.?

10. How many minutes from 15 minutes before 10 o'clock to 15 minutes past 11 o'clock?

11. James has 1lb. 8oz. of candy, and Edwin has $\frac{3}{4}$ as much. How much has Edwin?

SOLUTION.—1 pound 8 ounces is 24 ounces. Hence, James has 24 ounces, and since Edwin has $\frac{3}{4}$ as much, he has $\frac{3}{4}$ of 24 ounces, or 18 ounces, which is 1 pound 2 ounces.

12. If you have 1 pound 9 ounces of gold, and should sell $\frac{3}{7}$ of it, how much would you have left?

13. At $2 a hundred-weight, what will $\frac{4}{9}$ of 2 tons 5 hundred-weight of hay cost?

14. If you should gather in one day 3 quarts 1 pint of berries, and the next day 5 quarts 1 pint, how much would they be worth, at 12 cents a quart?

15. How many months old is a boy who has lived 11 years less 8 months?

16. Jason has 1 pound 6 ounces of candy, and his brother has 3 times as much. How much more than Jason has his brother?

17. My granary has two bins; one will hold 36 bushels 1 peck, and the other one fifth as much. How much less does one contain than the other?

WRITTEN EXERCISES.

372.—1. How many steps, of 3 feet each, must you take in walking 1 mile 6 rods?

2. How many miles has a man travelled who has gone 1793 steps of 3 feet each?

3. I have 2 plots of ground, each containing 1A. 120P. Into how many lots, of 20 square rods each, can the plots be cut?

4. A planet has moved in a given time 17° 30′. How many seconds has it moved?

5. At 90 cents a cord foot, how many cords of wood can be bought for $16.20?

6. I have 2 pounds of silver. How many spoons, weighing 1oz. 9pwt. each, can be made from it, and how many pennyweights will remain?

7. How many days, of 8 hours each, do you waste in a term of 12 weeks, each week containing 5 school days, if you are idle 2 hours each school day?

8. For how much will 4bu. 2pk. of strawberries sell, if put into quart boxes and retailed at 30 cents a box?

9. What will 4 casks of molasses, each containing 84gal. 3qt., sell for, at 15 cents a quart?

10. How many loads, each containing 1cd. 2 cd. ft., can be taken from a range of wood containing 25cd. 6cd. ft.?

11. If a train of cars can move in 1 hour, 25mi. 80rd., in what time can it move 250mi. 160rd.?

12. How many kegs, each containing 5gal. 3qt. 1pt., can be filled from a cask containing 58gal. 3qt.?

---

373. TEST QUESTIONS.—1. What is a denomination? What are the denominations of linear measure? Of surface measure? Of cubic measure?

2. What are the measures of lines, surfaces or solids called? Measures of extension. Recite the table of linear measures. The table of surface measures. The table of cubic measures.

3. What are the denominations of liquid measure? Of dry measure? What are the measures of liquids, or of grain, fruits and like articles, called? Measures of capacity. Recite the table of liquid measures. Of dry measures.

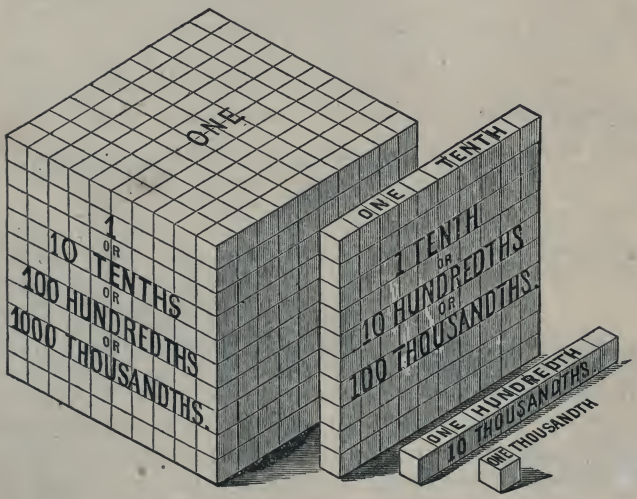
4. What are the denominations of Avoirdupois Weight? Of Troy Weight? What are these called? Measures of weight. Recite the table of Avoirdupois Weight. Of Troy Weight.

5. What are the denominations of circular measure? Recite the table. What are the denominations of the measure of time? Recite the table.

6. How does reduction descending differ from reduction ascending? Compound addition from compound subtraction? Compound multiplication from compound division?

SECTION XXXVI.

NOTATION AND NUMERATION OF DECIMALS.



374.—1. If a block of wood be divided into ten equal parts, what is 1 of the parts called? What are 2 of the parts called? 3 of the parts? 7 of the parts?

2. If one tenth of a block of wood be divided into ten equal parts, what is 1 of the parts called? What are 2 of the parts called? 3 of the parts? 7 of the parts?

3. What part of 1 is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{3}{10}$ of $\frac{1}{10}$? $\frac{7}{10}$ of $\frac{1}{10}$?

4. What part of one tenth is one hundredth? How many hundredths is one tenth?

5. If one hundredth of a block of wood be divided into ten equal parts, what is 1 of the parts called? What are 2 of the parts called? 3 of the parts? 7 of the parts?

6. What part of 1 is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{3}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$?

7. What part of one hundredth is one thousandth? How many thousandths is one hundredth?

DEFINITIONS.

375. A **Decimal Fraction,** or **Decimal,** is a number of *tenths, hundredths, thousandths,* etc., expressed by placing a point (.) before the numerator and omitting the denominator.

Thus, .3 expresses 3 tenths, or $\frac{3}{10}$; and .03 expresses 3 hundredths, or $\frac{3}{10\,0}$.

376. The point is called the *Decimal Point,* and in a numerical expression separates the decimal from the order of ones or units.

Thus, 3.003 expresses 3 ones and 3 thousandths, or $3\frac{3}{10\,0\,0}$.

377. The first order at the right of the decimal point is *tenths,* the second *hundredths,* the third *thousandths,* etc., as shown in the following

TABLE.

Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Ones.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
6	5	4	3	2	1	.	2	3	4	5	6	7
INTEGERS.							DECIMALS.					

A **Mixed Decimal** consists of an integer and a decimal.

Thus, 15.165, which is read *fifteen ones and one hundred sixty-five thousandths*, or *fifteen and one hundred sixty-five thousandths*, is a mixed decimal.

378. Principles.—1. *Ten of any decimal order are always equal to one of the next higher order.*

2. *The denominator of a decimal is* 1, *with as many ciphers annexed as there are orders in the decimal.*

WRITTEN EXERCISES.

379.—1. Write and read .308.

SOLUTION.—.308 = 3 tenths 0 hundredths 8 thousandths; or, .3 = 30 hundredths = 300 thousandths; and 300 thousandths + 8 thousandths = 308 thousandths. Hence, .308 is read *308 thousandths.*

2. Write and read 61.25.

SOLUTION.—61.25 = 61 ones and decimal .25; .25 = 2 tenths 5 hundredths; or, since .2 = 20 hundredths, .25 = 20 hundreths + 5 hundredths = 25 hundredths. Hence, 61.25 is read *61 ones and 25 hundredths*; or *61 and 25 hundredths.*

3. Write and read .3165.

4. Write and read 515.006.

5. Write three hundred five thousandths.

SOLUTION.—305 thousandths = 30 hundredths and 5 thousandths; 30 hundredths = 3 tenths and 0 hundredths. Hence, 305 thousandths = 3 tenths 0 hundredths 5 thousandths = .305.

6. Write one thousand three hundred thirty-one ten-thousandths.

7. Write eleven thousand eleven, and eleven thousandths.

380. Rules for Reading and Writing Decimals.—*Read a decimal as though it were an integer, giving it the name of its right-hand order.*

Write a decimal as though it were an integer, and place the decimal point so that each figure shall stand in its proper order, marking the absence of units of any order by a cipher.

PROBLEMS.

381. Write and read—

1. .347	4. .138	7. 14.005
2. .1405	5. .10065	8. 9.093
3. .0072	6. .000002	9. 1.6444

Write in the decimal form—

10. $\frac{5}{10}$.	13. $\frac{17}{1000}$.	16. $14\frac{5}{100}$.
11. $\frac{31}{100}$.	14. $\frac{434}{1000}$.	17. $9\frac{3}{1000}$.
12. $\frac{4}{100}$.	15. $\frac{1111}{10000}$.	18. $1\frac{6444}{100000}$.

19. Ninety-eight hundredths.

20. One hundred twenty-five thousandths.

21. One thousand eighteen ten-thousandths.

22. Six, and seventy-five hundredths.

23. Two thousand four hundred sixty-two, and seven tenths.

24. Four hundred, and forty-four thousandths.

25. Twenty-four, and twenty-four millionths.

26. Thirty-five thousand three hundred fifty-one, and twenty-one hundredths.

27. One million two hundred thousand thirty, and one millionth.

SECTION XXXVII.

ADDITION AND SUBTRACTION OF DECIMALS.

382.—1. What is the sum of 3 tenths and 5 tenths?

2. What is the sum of 7 tenths and 8 tenths?

SOLUTION.—7 tenths and 8 tenths are 15 tenths; and since 10 tenths are equal to 1, 15 tenths are 1 and 5 tenths, or 1.5. Hence, the sum of 7 tenths and 8 tenths is 1.5.

3. What is the sum of 12 hundredths and 11 hundredths?

4. If you pay 25 hundredths of a dollar for a slate, and 10 hundredths of a dollar for some paper, what part of a dollar do you pay for the whole?

5. If you pay 25 hundredths of a dollar for a slate, and 10 hundredths of a dollar for some paper, how much more does the slate cost than the paper?

6. If you have 75 hundredths of a bushel of chestnuts, and your brother has 15 hundredths of a bushel, how many more have you than your brother?

383. Principle.—*Decimals which express like parts of a unit may be added or subtracted like integers.*

WRITTEN EXERCISES.

384.—1. Add 13.634, 35.423 and 8.56.

$$\begin{array}{r} 13.634 \\ 35.423 \\ 8.56 \\ \hline 57.617 \end{array}$$

SOLUTION.—Write the numbers so that figures of the same order shall stand in the same column. Add as in addition of integers, which gives 57.617, the result required.

2. Add .315, 17.563 and 63.05.

3. From 963.75 subtract 585.125.

963.750
585.125
378.625

SOLUTION.—Write the numbers so that figures of the same order shall stand in the same column. Subtract as in subtraction of integers, which gives 378.625, the result required.

These numbers are prepared for subtracting by annexing a cipher to the minuend, which makes its decimal express thousandths, or like parts with the decimal of the subtrahend, without altering the value.

4. From 196.35 subtract 173.91.

5. From 73.007 subtract 68.75.

385. Rule for Addition and Subtraction of Decimals.— *Write the numbers so that figures of the same order shall stand in the same column.*

Add or subtract in the same manner as in integers, and place the decimal point at the left of the order of tenths in the result.

PROBLEMS.

386.—1. What is the sum of .81, 3.65, 4.5 and 7.315?

2. What is the sum of .9, 14.501, 6.75 and 19?

3. What is the sum of .125 + .62 +.437?

4. What is the difference between 41.634 and 7.595?

5. What is the difference between 18.5 and 9.995?

6. What is the difference between .735 of a ton and .598 of a ton?

7. What is the difference between 98 and .98?

8. Wilson is 63.125 years old, and Johnson is 58.75 years old. What is the sum of their ages?

9. Horatio has 125.675 thousand feet of box-boards, and his father 239.703 thousand feet. How many more has the one than the other?

10. A farmer received $69.875 for corn, $93.1875 for wheat, and $42.9375 for oats. How much did he receive for the whole?

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387. TEST QUESTIONS.—1. What is a decimal? A mixed decimal?

2. Recite the decimal orders from tenths to millionths. What principles of decimals are given?

3. How do you read decimals? How do you write decimals?

4. What is the principle for addition and subtraction of decimals? How do you add or subtract decimals?

SECTION XXXVIII.

MULTIPLICATION OF DECIMALS.

388.—1. How many tenths are 3 times 1 tenth? 3 times 3 tenths? 5 times 4 tenths?

2. How many ones are 5 times 4 tenths? 5 times 9 tenths?

SOLUTION.—5 times 9 tenths are 45 tenths; and since 10 tenths are 1, 45 tenths are 4 ones and 5 tenths, or 4.5.

3. How many hundredths are 3 times 1 hundredth? 3 times 3 hundredths? 5 times 4 hundredths?

4. How many tenths are 5 times 4 hundredths?

5. When tenths are multiplied by ones, what is the denominator of the product? When hundredths are multiplied by ones, what is the denominator?

6. What is the product of 1 tenth by 1 tenth, or of $\frac{1}{10}$ by $\frac{1}{10}$? Of 3 tenths by 2 tenths, or $\frac{3}{10}$ by $\frac{2}{10}$?

7. What is the product of 1 hundredth by 1 tenth, or $\frac{1}{100}$ by $\frac{1}{10}$? Of 3 hundredths by 2 tenths, or $\frac{3}{100}$ by $\frac{2}{10}$?

8. When tenths are multiplied by tenths, what is the denominator of the product? When hundredths are multiplied by tenths?

9. How many decimal orders will there be in the product, decimally expressed, if you multiply $\frac{1}{10}$ by 3? $\frac{1}{10}$ by $\frac{3}{10}$? $\frac{1}{100}$ by 3? $\frac{1}{100}$ by $\frac{3}{10}$?

389. Principle.—*The number of decimal orders in the product is equal to the number of decimal orders in both factors.*

WRITTEN EXERCISES.

390.—1. What is the product of .49 by 6?

$$\begin{array}{r} .49 \\ 6 \\ \hline 2.94 \end{array}$$

SOLUTION.—6 times $\frac{49}{100}$ are $\frac{49 \times 6}{100} = \frac{294}{100} = 2\frac{94}{100}$, or decimally expressed, 2.94.

2. What is the product of .49 by .6?

$$\begin{array}{r} .49 \\ .6 \\ \hline .294 \end{array}$$

SOLUTION.—$\frac{6}{10}$ times $\frac{49}{100}$ are $\frac{49}{100} \times \frac{6}{10} = \frac{49 \times 6}{100 \times 10} = \frac{294}{1000}$, or, decimally expressed, .294.

3. What is the product of .573 by 5?

4. What is the product of .75 by .06?

391. Rule for Multiplication of Decimals.—*Multiply as in integers, and point off as many orders for decimals in the product as there are orders of decimals in both factors.*

If there are not as many figures in the product as decimal orders required, supply the deficiency by prefixing ciphers.

PROBLEMS.

392. Multiply—

1. 35 by .5	5. 4.3 by .7	9. 4.06 by 1.2
2. 35 by .05	6. .43 by .07	10. 4.06 by .12
3. 3.5 by 5.	7. 4.3 by 7.	11. .406 by 12.
4. .35 by 5.	8. 43 by .7	12. 40.6 by 1.2

13. The distance to a certain place is .75 of a mile; to another place it is 10 times as far. What is the distance to the latter place?

Since the multiplier is 10, the product may be found at once by removing the decimal point in the multiplicand one order to the right.

14. The standard bushel contains 2150.42 cubic inches. What is the cubic capacity of a bin containing 100 bushels?

15. What is the product of .00365 by 1000?

16. What will 10.25 yards of cloth cost, at $3.40 per yard?

SECTION XXXIX.

DIVISION OF DECIMALS.

393.—1. How many times 3 tenths in 9 tenths?

2. How many times 3 hundredths in 9 tenths?

SOLUTION.—9 tenths are 90 hundredths; 3 hundredths are contained in 90 hundredths 30 times.

3. How many times 2 hundredths in 8 tenths? In 6 tenths?

4. What is the quotient of 9 tenths by 3 tenths, or $\frac{9}{10}$ by $\frac{3}{10}$? Of $\frac{8}{10}$ by $\frac{2}{10}$?

5. What is the quotient of 8 tenths by 2 hundredths, or $\frac{8}{10}$ by $\frac{2}{100}$? Of $\frac{6}{10}$ by $\frac{2}{100}$?

6. What is the quotient of 5 tenths by 25 hundredths, or .5 by .25? Of 1.2 by .12?

7. What is the quotient of 4.2 by 6, or what is one sixth of 4.2?

SOLUTION.—4.2 is 42 tenths; and one sixth of 42 tenths is 7 tenths.

8. What is the quotient of .42 by 6? Of .42 by .06? Of 6 by .6?

9. When .42 is divided by 6, how many more orders of decimals has the dividend than the divisor?

10. When .42 is divided by 6, how many orders of decimals has the quotient?

394. Principle.—*The number of decimal orders in the quotient is as many as there are in the dividend less the number in the divisor.*

WRITTEN EXERCISES.

395.—1. How many times 6 in 2.94?

$6)\underline{2.94}$
$.49$

SOLUTION.—2.94 is 294 hundredths, and one sixth of 294 hundredths is 49 hundredths, or .49.

2. How many times .6 in .294?

$.6)\underline{.294}$
$.49$

SOLUTION.—.6 is the same as $\frac{1}{10}$ of 6; 6 is contained in .294 one sixth of .294, or .049, times; and $\frac{1}{10}$ of 6 must be contained 10 times .049 times, or .49 times.

3. How many times 9 in 28.35?

4. How many times .07 in .0938?

396. Rule for Division of Decimals.—*Divide as in integers, and point off in the quotient as many decimal orders as those in the dividend exceed in number those in the divisor.*

If there are not as many figures in the quotient as the number of decimal orders required, supply the deficiency by prefixing ciphers.

PROBLEMS.

397. Divide—

1. 6.15 by 5.	5. .0075 by .15	9. 32.76 by 78.
2. 30.1 by .7	6. 99 by .99	10. 60 by .8
3. .825 by .25	7. .99 by 99.	11. 99 by 9.9
4. 4.06 by 1.2	8. 9.9 by .99	12. .0738 by .6

13. What is the quotient of 75.6 divided by 100?

Since the divisor is 100, the quotient is obtained at once by removing the decimal point in the dividend two orders to the left.

14. What is the quotient of 31.45 divided by 1000?

15. What is the quotient of .905 divided by 10000?

16. What is the quotient of 23 by .3 to 2 orders of decimals?

$.3)\overline{23.000}$

$76.66+$

SOLUTION.—Annex ciphers as decimal orders, and continue the division till a quotient is obtained with the required number of decimal orders, which is 76.66 +.

The sign + is annexed to the result to indicate that the division is incomplete, and could have been carried farther.

17. What is the quotient of .025 by .41 to 4 orders of decimals?

18. How many yards of cloth at $.17 a yard can be purchased for $2.635?

SECTION XL.

REDUCTION OF DECIMALS.

CASE I.

Decimals Reduced to Common Fractions.

398.—1. What does .5 express? How many halves in $\frac{5}{10}$?

2. What does .6 express? How many fifths in $\frac{6}{10}$?

3. What does .75 express? How many fourths in $\frac{75}{100}$?

4. What does .45 express? How many twentieths in $\frac{45}{100}$?

5. What does .045 express? How many hundredths in $\frac{45}{1000}$?

WRITTEN EXERCISES.

399.—1. Reduce .25 to a common fraction.

$.25 = \frac{25}{100} = \frac{5}{20} = \frac{1}{4}$

SOLUTION.—.25 may be expressed in the form of a common fraction by omitting the decimal point and writing the denominator, which gives $\frac{25}{100}$.

$\frac{25}{100}$ reduced to its lowest terms is $\frac{1}{4}$, which is the fraction required.

2. Reduce .05 to a common fraction.

3. Reduce .056 to a common fraction.

4. Reduce .125 to a common fraction.

5. Reduce .625 to a common fraction.

400. Rule for Reduction of Decimals to Common Fractions.—*Omit the decimal point; write the denominator under the given numerator, and reduce the fraction to its lowest terms.*

PROBLEMS.

401. Reduce to common fractions in lowest terms—

1. .85	4. .016	7. .096
2. .125	5. .625	8. .008
3. .025	6. .0125	9. 1.275

CASE II.

Common Fractions Reduced to Decimals.

402.—1. How many tenths in $\frac{1}{2}$? In $\frac{3}{5}$? In $\frac{4}{5}$?
2. How many hundredths in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{2}{5}$?
3. How many hundredths in $\frac{3}{4}$? In $\frac{4}{20}$? In $\frac{3}{25}$?

WRITTEN EXERCISES.

403.—1. Reduce $\frac{3}{8}$ to a decimal.

$$8\overline{)3.000}\quad .375$$

SOLUTION.—$\frac{3}{8}$ is $\frac{1}{8}$ of 3.

3 is 30 tenths, or 3.0; and $\frac{1}{8}$ of 30 tenths is 3 tenths, or .3, with 6 tenths remaining.

6 tenths are 60 hundredths; and $\frac{1}{8}$ of 60 hundredths is 7 hundredths, or .07, with 4 hundredths remaining.

4 hundredths are 40 thousandths; and $\frac{1}{8}$ of 40 thousandths is 5 thousandths, or .005.

The result is 3 tenths 7 hundredths 5 thousandths, or .375, which is the decimal required.

2. Reduce $\frac{3}{4}$ to a decimal.
3. Reduce $\frac{2}{25}$ to a decimal.

404. Rule for Reduction of Common Fractions to Decimals.—*Reduce the numerator to tenths, hundredths, etc., by annexing ciphers; divide this result by the denominator, and point off as many orders for decimals in the quotient as there are ciphers annexed.*

PROBLEMS.

405. Reduce to decimals—

1. $\frac{17}{20}$.	4. $\frac{2}{125}$.	7. $\frac{12}{125}$.
2. $\frac{1}{8}$.	5. $\frac{5}{8}$.	8. $\frac{1}{125}$.
3. $\frac{1}{40}$.	6. $\frac{1}{16}$.	9. $1\frac{11}{40}$.

10. Reduce $\frac{3}{11}$ to a decimal of 3 orders.

11)3.000
.272+

SOLUTION.—Reduce the 3 to thousandths by annexing three ciphers and divide by 11. Denote that the decimal, with the required orders, is not exact by annexing the sign +.

11. Reduce $\frac{5}{13}$ to a decimal of 4 orders.

---

406. TEST QUESTIONS.—1. What is a principle of multiplication of decimals? How do you multiply in decimals?

2. What is a principle of division of decimals? How do you divide in decimals?

3. How do you reduce decimals to common fractions? Common fractions to decimals?

SECTION XLI.

REVIEW OF DECIMALS.

WRITTEN EXERCISES.

407.—1. Express .0375 as a common fraction.

2. Express $\frac{17}{10000}$ as a decimal.

3. What part of a dollar is \$.125?

4. What is the value of $\frac{7}{19}$, expressed as a decimal of 3 orders?

5. A farmer sold 15 cows for \$828.75; how much did he get for each?

6. On one car there are 15.365 thousand feet of boards, and on another 16.491 thousand feet. How many thousand feet are there on both cars?

7. By the census of 1870, there were in Rhode Island, 206 persons to a square mile; in Massachusetts, 185.6 persons to a square mile. How many more persons to a square mile were there in Rhode Island than in Massachusetts?

8. What is the value of 9765 pounds of fish, at $5.20 per hundred?

9. What must be paid for 5305 feet of boards, at $11.40 per thousand feet?

10. How many feet of boards can be bought for $60.477, at $11.40 per thousand feet?

11. How much is .005 divided by .00005?

12. A cubic foot of white oak weighs 42.937 pounds; what is the weight of 50 cubic feet?

SECTION XLII.

NOTATION OF PERCENTAGE.

408.—1. How much is $\frac{1}{100}$ of 100 pounds? $\frac{2}{100}$ of 100 pounds? $\frac{5}{100}$ of 100 pounds?

2. How much is $\frac{3}{100}$ of 400 yards? $\frac{7}{100}$ of 400 yards? $\frac{9}{100}$ of 400 yards?

3. What is a per cent. of a number? A **per cent.** of a number is a number of hundredths of that number.

4. How many per cent. of a number is $\frac{1}{100}$? Is $\frac{3}{100}$?

5. To take 4 per cent. of a number, is to take how many hundredths of the number?

6. What part of a number is 2 per cent. of a number?

SOLUTION.—2 per cent. is $\frac{2}{100}$, or $\frac{1}{50}$; hence, 2 per cent. of a number is $\frac{1}{50}$ of the number.

7. What part of a number is 4 per cent. of it? 5 per cent. of it? 10 per cent. of it?

8. What part of a number is 20 per cent. of it? 40 per cent. of it? 50 per cent. of it?

9. What per cent. of a number is expressed by $\frac{30}{100}$? By $\frac{75}{100}$? By $\frac{125}{100}$?

10. What per cent. of a number is .06 of it? Is .30 of it? Is .75 of it? Is 1.25 of it?

DEFINITIONS.

409. Any **Per Cent.** of a number is so many hundredths of the number.

The term per cent. is a contraction of *per centum*, which means *by the hundred.*

410. The **Rate per cent.** is the number denoting the number of hundredths.

411. The **Sign** of per cent. is %, and is read *per cent.* Thus,

$\frac{1}{100}$, or .01, may be written 1%, and read *one per cent.*

$\frac{6}{100}$, or .06, " " 6%, " *six per cent.*

$\frac{4\frac{1}{2}}{100}$, or .04$\frac{1}{2}$, " " 4½%, " *four and one half per cent.*

$\frac{115}{100}$, or 1.15, " " 115%, " *one hundred fifteen per cent.*

412. The **Base** is the number or quantity of which the per cent. is computed.

413. **Percentage** is the process of computing by hundredths.

The term *Percentage* is applied, also, to that part of the base which corresponds to the rate per cent.

WRITTEN EXERCISES.

414. Express decimally—

1. 3%.	5. $4\frac{3}{4}$%.	9. 45%.
2. 4%.	6. $9\frac{1}{2}$%.	10. 67%.
3. 7%.	7. $17\frac{1}{4}$%.	11. 125%.
4. 9%.	8. $7\frac{1}{8}$%.	12. 135%.

SECTION XLIII.

CASES IN PERCENTAGE.

CASE I.

Base and Rate Per Cent. given, to find the Percentage.

415.—1. What part of a number is 4% of it?

SOLUTION.—4% is $\frac{4}{100}$, or $\frac{1}{25}$; hence, 4% of a number is $\frac{1}{25}$ of the number.

2. What part of a number is 15% of it? 30% of it?

3. What part of a number is 20% of it? 12% of it? 50% of it?

4. What is the simplest form of 20%?

SOLUTION.—20% is $\frac{20}{100}$, which in its simplest form is $\frac{1}{5}$.

5. What is the simplest form of 50%? Of 60%? Of 80%?

6. What is the simplest form of $12\frac{1}{2}\%$? Of $16\frac{2}{3}\%$? Of $33\frac{1}{3}\%$?

7. A farmer had 20 cows, and sold 10% of them. How many did he sell?

8. If from a flock of sheep 5% of them should be taken, what per cent. would remain?

9. A man bought a horse for $100, and sold him at a profit of 25%. For what sum did he sell him?

10. If you buy a horse for $100, and sell him at a loss of 20%, how much do you get for him?

WRITTEN EXERCISES.

416.—1. What is 7% of 9546 yards?

9546 yds.
.07
668.22 yds.

SOLUTION.—Since 7% is .07, 7% of 9546 yards is .07 of 9546 yards, which is 668.22 yards, the percentage required.

2. What is 5% of 860 pounds?

3. What is 12% of 750 men?

417. Rule for Finding any Per Cent. of a Number.—*Multiply the given number by the rate per cent. expressed decimally.*

PROBLEMS.

418. How much is—

1. 4% of $61.50?
2. $3\frac{1}{2}\%$ of $974?
3. $7\frac{1}{4}\%$ of 160yd.?
4. 25% of 530rd.?
5. $1\frac{3}{4}\%$ of 9160?
6. $3\frac{1}{4}\%$ of 850?
7. 19% of 562?
8. 12% of 880?
9. 10% of $750?
10. 8% of 450 men?
11. 15% of 560cwt.?
12. 60% of 725lb.?

13. If you earn in a year $300, and spend 45% of that sum, how much do you spend?

14. What is a merchant's commission for selling goods to the amount of $7500, at $2\frac{1}{2}\%$?

15. Of a cargo of corn, consisting of 2560 bushels, 15% was damaged. How much was not damaged?

16. If a town having 9540 inhabitants should increase in population 20%, what would be the number of its inhabitants?

CASE II.

Base and Percentage given, to find the Rate Per Cent.

419.—1. What per cent. of a number is $\frac{1}{5}$ of it?

2. What part of 5 is 2? What per cent. of 1 is $\frac{2}{5}$?

3. I spent $\frac{1}{4}$ of my money. What per cent. of it did I spend?

4. I had $20, and spent $5. What per cent. of my money did I spend?

SOLUTION.—I had $20, and spent $5; hence, I must have spent $\frac{5}{20}$, or $\frac{1}{4}$ of my money; and $\frac{1}{4}$ equals $\frac{25}{100}$, or 25%.

5. What per cent. of $10 is $2? Of $45 is 30?

6. 8 days are what per cent. of 20 days? Of 48 days?

7. I received $3 for collecting $60. What per cent. did I receive?

8. A farmer bought a cart for $50, and sold it for $60. What per cent. did he gain?

9. James bought a knife for $.60, and sold it for $.48. What per cent. was the loss?

10. What per cent. does a merchant gain who buys flour at $8 per barrel, and sells it at the rate of 3 barrels for $30?

WRITTEN EXERCISES.

420.—1. What per cent. of 400 is 24?

$400)24.00(.06=6\%$
$\underline{24.00}$

SOLUTION.—Since 24 is $\frac{24}{400}$ of 400, it is .06, or 6%, of 400, which is the rate per cent. required.

2. What per cent. of 520 is 26?

3. What per cent. of 95 bushels are 3.6 bushels?

421. Rule for finding the Rate Per Cent. that a given Percentage is of the Base.—*Divide the percentage by the base, extending the quotient to hundredths.*

PROBLEMS.

422. What per cent. of—

1. 54 yards are 9 yards?
2. 125 men are 8 men?
3. $650 are $39.25?
4. 530rd. are 132½rd.?
5. 9160 is 160.3?
6. 400 is 28?
7. 560 is 106.4?
8. 725 is 435?
9. $756 are $75?
10. 450 men are 36 men?
11. 560cwt. are 84cwt.?
12. 725lb. are 435lb.?

13. If you earn $300, and spend $135 of it, what rate per cent. of the sum earned is the sum spent?

14. A house, which cost $5670, was sold for $283.50 above the cost. What was the rate per cent. of gain?

15. Of an army of 45450 men, 12626 men were lost in battle. What was the rate per cent. of loss?

16. If a horse, which cost $250, was sold for $100 above cost, what was the rate per cent. gained?

17. A certain town, which had 9540 inhabitants, has increased by 1908 persons. What is the rate per cent. of increase?

SECTION LIV.

INTEREST.

423.—1. When the allowance for the use of money is 6%, how many cents is it for \$1, or 100 cents? How much is it for \$100?

2. When the allowance for the use of \$1 is 6 cents, what is the rate per cent.? When it is 5 cents?

3. When the allowance for the use of \$100 for 1 year is \$6, what is the yearly rate per cent.? When it is \$5, what is the yearly rate per cent.?

4. When the yearly allowance for the use of money is at the rate of 6%, what is the allowance for the use of \$5 for 3 years?

SOLUTION.—The allowance for 1 year at 6% is $\frac{6}{100}$ of the base; hence, the allowance for the use of \$5 for 1 year must be $\frac{6}{100}$ of \$5, or \$.30, and for the use of \$5 for 3 years must be 3 times \$.30, or \$.90.

5. What is the allowance for the use of $8 for 4 years, at a yearly rate of 7%?

6. What is the allowance for the use of $25 for 5 years, at a yearly rate of 4%?

7. What is the allowance for the use of $50 for 4 months, at a yearly rate of 6%?

SOLUTION.—The allowance for the use of $50 for 1 year at 6 per cent. is $3; hence, for 4 months, or $\frac{1}{3}$ of a year, it must be $\frac{1}{3}$ of $3, or $1.

8. What is the allowance for the use of $40 for 3 months, at a yearly rate of 7%?

9. What is the allowance for the use of $90 for 2 months, at a yearly rate of 6%?

10. In computing the allowance for the use of money, a month is commonly regarded as how many days?

Ans. 30 days.

11. What is the allowance for the use of $200 for 1 month 20 days, at a yearly rate of 6%?

SOLUTION.—The allowance for the use of $200 for 1 year is $12; hence, for 1 month, or $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of $12, or $1, and for 20 days, or $\frac{2}{3}$ of a month, it is $\frac{2}{3}$ of $1, or $.66$\frac{2}{3}$; $1 and $.66$\frac{2}{3}$ is 1.66\frac{2}{3}$, the allowance required.

12. What is the allowance for the use of $100 for 2 months 15 days, at a yearly rate of 8%?

13. What is the allowance for the use of $300 for 1 month 10 days, at a yearly rate of 5%?

14. What is the allowance for the use of $120 for 1 year 4 months 6 days, at a yearly rate of 5%?

15. What is the allowance for the use of $200 for 2 years 5 months 15 days, at a yearly rate of 12%? At a yearly rate of 16$\frac{2}{3}$%?

DEFINITIONS.

424. **Interest** is an allowance for the use of money or its equivalent.

425. The **Principal** is the sum for the use of which interest is allowed.

426. The **Rate of Interest** is the rate per cent. of the principal allowed for its use for one year.

427. The **Amount** is the sum of the principal and interest.

WRITTEN EXERCISES.

428.—1. What is the interest of \$345.50 for 3 years 3 months, at 6%?

Principal,	\$345.50
Rate,	.06
Int. for 1y.,	\$20.7300
	3
Int. for 3y.,	\$62.19
Int. for 3mo., or $\frac{1}{4}$y.,	$5.18\frac{1}{4}$
Int. for 3y. 3mo.,	$\$67.37\frac{1}{4}$

SOLUTION.—The interest of \$345.50 for 1 year is .06 of that sum, or \$20.73; hence, for 3 years the interest is 3 times \$20.73, or \$62.19.

The interest of \$345.50 for 1 year is \$20.73; hence, for 3 months, or $\frac{1}{4}$ of a year, it is $\frac{1}{4}$ of \$20.73, or $\$5.18\frac{1}{4}$. The interest for 3 years 3 months is $\$62.19 + \$5.18\frac{1}{4}$, or $\$67.37\frac{1}{4}$.

2. What is the interest of \$500 for 5 years 6 months, at 7%?

3. What is the interest of \$750 for 4 years 5 months, at 8%?

4. What is the interest of \$840 for 3 years 4 months, at 6%?

5. What is the interest of \$920 for 4 years 2 months, at 5%?

6. What is the amount of $450 for 7 months 20 days, at 4%?

Principal,	$450
Rate,	.04
Int. for 1y.,	$18.00
Int. for 6mo., or $\frac{1}{2}$y.,	$9.00
Int. for 1mo., or $\frac{1}{6}$ of 6mo.,	1.50
Int. for 20d., or $\frac{2}{3}$mo.,	1.00
Int. for 7mo. 20d.,	$11.50
Principal,	450
Amount,	$461.50

SOLUTION.—The interest of $450 for 1 year is .04 of the principal, or $18; hence, for 6mo., or $\frac{1}{2}$ year, it is $\frac{1}{2}$ of $18, or $9; for 1mo., or $\frac{1}{6}$ of 6mo., it is $\frac{1}{6}$ of $9, or $1.50; and for 20d., or $\frac{2}{3}$mo., it is $\frac{2}{3}$ of $1.50, or $1; hence, for 7mo. 20d. it is $9 + $1.50 + $1, or $11.50.

The amount is the sum of the principal and interest, $450 + $11.50, which is $461.50.

7. What is the amount of $600 for 5 months 12 days, at 7%?

8. What is the amount of $720 for 2 months 3 days, at 6%?

9. What is the interest of $5000 for 25 days, at 8%?

10. What is the interest of $4200 for 18 days, at 6%?

429. Rules for Computing Interest.—*Multiply the principal by the rate of interest expressed decimally, and that product by the time expressed in years.*

If there be months or days, find the interest for the number of months, by taking fractional parts of a year's interest; and for the number of days, by taking fractional parts of a month's interest.

To find the amount, add the principal and interest.

PROBLEMS.

430.—What is the interest of—

1. $1800 for 3 years 9 months, at 7%?
2. $34.50 for 1 year 6 months, at 6%?
3. $75.50 for 6 years 11 months, at 8%?
4. $5600 for 4 years 5 months, at 5%?
5. $8000 for 2 years 2 months, at 4%?
6. $10000 for 15 years 1 month, at $3\frac{1}{2}$%?
7. $240 for 10 months 6 days, at 9%?
8. $6450 for 3 months 3 days, at 10%?
9. $65.40 for 1 month 24 days, at 6%?
10. What is the amount of $725 for 1 year 6 months 10 days, at 8%?
11. What is the amount of $830 for 2 years 3 months 27 days, at 7%?
12. What is the amount of $300 for 18 days, at 6%?
13. What is the interest of $407.60 from Jan. 1, 1869, to April 1, 1871, at 6%?
14. What is the amount of $1000 from May 15, 1870, to October 18, 1871, at 7%?
15. What is the amount of $1200 from February 1, 1871, to November 1, 1872, at 8%?

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431. TEST QUESTIONS.—1. What is any per cent. of a number? What is the rate per cent.? The base? What is percentage?

2. How do you find any per cent. of a number? The rate per cent. a given percentage is of a base?

3. What is interest? The principal? The rate of interest? The amount? How do you compute interest?

SECTION XLV.

REVIEW OF PERCENTAGE.

432.—1. What is a broker's charge, at $\frac{1}{4}\%$, for buying stocks amounting to $400?

2. An agent sells goods to the amount of $500, and reserves $2\frac{1}{2}\%$ of the amount for his services. How much did he receive?

3. I bought broadcloth for $5 per yard, and sold it at 20% above cost. How much did I gain per yard?

4. I bought cloth at $5 per yard, and sold it so as to gain $1 per yard. What rate per cent. was the gain?

5. I bought a watch for $60, and sold it at a loss of 10%. How much did I get for it?

6. At what price must I sell a watch that cost me $50 to gain 20%?

7. If I sell a watch which cost me $60, at a loss of 20%, how much do I get for it?

8. How many dollars are $20 less 5% of $20?

9. I sold a carriage which cost me $80 for $60. What was the rate per cent. of loss?

10. If you sell what cost you $6 for $8, what is the rate per cent. of gain?

11. What is the interest of $200 for 3 years 6 months, at 6%?

12. What is the interest of $400 for 15 days, at 6%?

13. What is the amount of $300 for 4 months at 7%?

14. What is the amount of $50 for 6 months, at 8%?

15. What is the interest of $600 for 5 years 9 months, at 6%?

WRITTEN EXERCISES.

433.—1. How much must be paid for insuring a house for $1500, at $2\frac{1}{2}\%$?

2. What is the interest of $524 for 3 months 12 days, at $4\frac{1}{2}\%$?

3. If you buy wheat at $1.80 per bushel, and sell it so as to gain 5%, what do you get per bushel for it?

4. A school-house which cost $7500 is insured for $66\frac{2}{3}\%$ of its cost. For how much is it insured?

5. What is the amount of $1004.45 for 1 year 6 months 6 days, at 6%?

6. If you give your note for $800, and pay it 2 years 8 months afterward, with interest at 7%, what sum will be required?

7. If you should have a note for $1200, which is payable in 2 months 3 days, without interest, and in borrowing money upon it you should allow interest in advance, for the time, at 6%, how much would that interest be?

8. I bought a village lot for $500, and sold it for $550. What rate per cent. was my gain?

9. If I should ask $550 for a village lot, but should fall upon the price 10%, what would then be my price?

10. I bought a horse for $250; at what price must I sell him to gain 20%?

11. What sum will be required to pay a note for $1540, which has been on interest at 6% for 3 years 4 months 24 days?

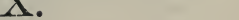
12. I bought a farm, May 16, 1870, for $4670, and paid for it, October 22, 1871, with interest at 8%. How much did I pay?

APPENDIX.

RECTANGULAR MEASUREMENTS.

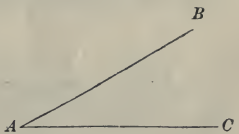
434. A **Line** is that which has only length.

435. A **Straight Line** is a line that has only one direction.



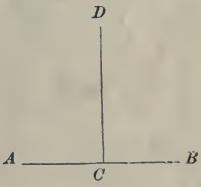
Thus, the line $A\ B$ is a straight line.

436. An **Angle** is the difference of direction of two lines that meet.



Thus, the lines AB and AC, meeting at A, form the angle CAB.

437. A **Perpendicular Line** is a straight line which meets another straight line so as to form two equal angles.



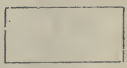
Thus, the line CD is a perpendicular line.

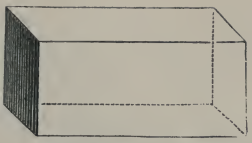
438. A **Right Angle** is an angle formed by two lines perpendicular to each other.

Thus, the angles ACD and DCB are each right angles.

439. A **Rectangle** is any figure which has four straight sides and four equal angles.

Thus, the figure in the margin, having four straight sides and four equal angles, is a rectangle.





440. A **Rectangular Solid** is any volume bounded by six rectangular faces.

Thus, the figure in the margin, having six rectangular faces, represents a rectangular solid.

CASE I.

Rectangular Surfaces.

441.—1. How many square feet are in a rectangular surface which is 3 feet long and 1 foot broad?

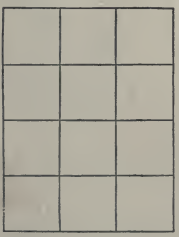
2. How many square inches are in a rectangular strip of paper which is 6 inches long and 1 inch wide?

3. How much will a rectangular board 9 feet long and 1 foot broad cost, at 3 cents per square foot?

DEFINITIONS.

442. The **Dimensions** of a rectangular surface are its length and breadth.

443. The **Area** of a rectangle is the extent of surface bounded by its two dimensions or sides.



Thus, the rectangle in the margin will be seen to contain 12 square inches, if it be supposed to be 4 inches long and 3 inches wide.

For, upon each inch of length there may be conceived to be 1 square inch, making a row of 4 square inches; and as there will be as many such rows as there are inches in the width, or 3 rows, the area of the rectangle must be 3 times 4 square inches, or 12 square inches.

WRITTEN EXERCISES.

444.—1. How many square feet of surface are there in a floor 18 feet long and $15\frac{1}{2}$ feet wide?

18 sq. ft. × 15½ = 279 sq. ft. SOLUTION.—A floor 18 feet long and 1 foot wide will contain 18 square feet of surface; and a floor 18 feet long and 15½ feet wide must contain 15½ times 18 square feet, or 279 square feet.

2. A rectangular garden 14 rods broad contains 280 square rods. How long is it?

280 ÷ 14 = 20, no. of rods in length. SOLUTION.—The area, 280 square rods, is the product of the length and breadth of the garden.

The length, then, must be the quotient arising from the division of the area, 280, by the breadth, 14, which is 20 rods.

3. A garden-walk has an area of 247 square feet and is 3¼ feet wide. What is its length?

4. A rectangular floor is 27 feet long and 15 feet wide. How many square yards of carpeting will be required to cover it?

445. Rules for Measurements of Rectangular Surfaces.—*1. Multiply the length by the width, and the product will denote the area.*

2. Divide the area by either of the dimensions, and the quotient will denote the other dimension.

PROBLEMS.

446.—1. A field is 40 rods long and 35 rods wide. What is its area?

2. The floor of a square room measures 43 feet on a side. What is its area?

3. I have a field containing 3698 square rods. Its length is 86 rods; what is its width?

4. How long must be a rectangular lot which is 16 rods wide to contain one and a half acres?

5. A piece of carpeting is 32 inches wide and 45 yards long. How many square yards of floor will it cover?

CASE II.

Rectangular Solids.

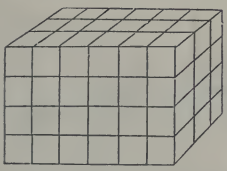
447.—1. How many cubic feet are in a rectangular piece of timber 12 feet long, 1 foot wide, and 1 foot thick?

2. How many cubic feet are in a rectangular beam 20 feet long, 1 foot wide, and 1 foot thick?

DEFINITIONS.

448. The **Dimensions** of a rectangular solid are its length, breadth, and thickness.

449. The **Cubic Contents** of a rectangular solid, or volume, is the extent bounded by its faces.



Thus, the rectangular solid, or volume, in the margin will be seen to contain 72 cubic inches, if it be supposed to be 6 inches long, 3 inches wide, and 4 inches thick.

For, upon each of the 18 square inches of the lower face there may be conceived to be 1 cubic inch, making a layer of 18 cubic inches; and as there will be as many such layers as there are inches of thickness, or 4, the contents of the volume must be 4 times 18 cubic inches, or 72 cubic inches.

WRITTEN EXERCISES.

450.—1. How many cubic feet are there in a rectangular piece of timber 16 feet long, 2 feet 6 inches broad, and 1 foot 3 inches thick?

$16\ cu.\ ft. \times 2\frac{1}{2} = 40\ cu.\ ft.$

$40\ cu.\ ft. \times 1\frac{1}{4} = 50\ cu.\ ft.$

SOLUTION.—2 ft. 6 in. $= 2\frac{1}{2}$ ft.; 1 ft. 3 in. $= 1\frac{1}{4}$ ft.

A piece of timber 16 feet long, 1 ft. broad, and 1 ft. thick will contain 16 cu. ft.; a piece 16 ft. long, $2\frac{1}{2}$ ft. broad, and 1 ft. thick must contain $2\frac{1}{2}$ times 16 cu. ft., or 40 cu. ft.; and a piece 16 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{4}$ ft. thick must contain $1\frac{1}{4}$ times 40 cu. ft., or 50 cu. ft.

2. A rectangular piece of timber whose cubic contents are 50 cubic feet is 16 feet long and $2\frac{1}{2}$ feet broad. What is its thickness?

$16 \times 2\frac{1}{2} = 40.$
$50 \div 40 = 1\frac{1}{4}.$

SOLUTION.—The cubic contents, 50 cubic feet, are the product of the length, breadth, and thickness.

The thickness, then, must be the quotient arising from the division of the cubic contents, 50 cu. ft., by the product of the given dimensions, 16 and $2\frac{1}{2}$, $= 40$, which gives $1\frac{1}{4}$ ft. as the thickness required.

3. How many cords of 4-foot wood in a range 28 feet in length and 6 feet in height?

$28 \times 4 \times 6 = 672$, *no. of cu. ft.*
$672 \div 16 = 42$, *no. of cd. ft.*
$42 \div 8 = 5\frac{1}{4}$, *no. of cords.*

SOLUTION.—The contents of the range equal $28 \times 4 \times 6 = 672$ cubic feet.

Since 16 cu. ft. are 1 cd. ft., there must be as many cd. ft. as there are times 16 cu. ft. in 672 cu. ft., or 42 cd. ft.

Since 8 cd. ft. are 1 cord, there must be as many cords as there are times 8 cd. ft. in 42 cd. ft., or $5\frac{1}{4}$ cords.

4. How many cord feet are in a load of wood 8 ft. long, 3 ft. 6 in. wide, and 4 ft. high?

451. Rules for Measurement of Rectangular Solids.—*1. Multiply the length, width, and thickness together, and the product will denote the cubic contents.*

2. Divide the cubic contents by the product of any two of the dimensions, and the quotient will denote the other dimension.

PROBLEMS.

452.—1. A bin is 8 feet long, 6 feet broad, and 4 feet 6 inches deep. How many cubic feet is its capacity?

2. A load of wood 8 feet long and 3 feet wide is piled 5 feet 4 inches high. How many cords are in the load?

3. A rectangular wall is 120 feet long, $2\frac{1}{2}$ feet wide, and 5 feet high. How many cubic feet does it contain?

4. How many cubic feet of earth must be removed in excavating a cellar 30 feet long, 22 feet wide, and 8 feet deep?

5. A rectangular block of marble containing 36 cubic feet is 8 feet long and 2 feet 3 inches thick. What is its width?

6. How much will it cost to remove a range of 4-foot wood which is 48 feet long and 6 feet high, at 80 cents a cord?

Capacity of Bins, Vats and Cisterns.

453. The **Standard Bushel** in the United States contains 2150.42 cubic inches.

Hence, a cubic foot, expressed in bushels, is equivalent to $\frac{1728}{2150.42} = .8036$, or about .8 of a bushel.

454. The **Standard Gallon,** liquid measure, contains 231 cubic inches.

Hence, a cubic foot, expressed in gallons, is equivalent to $\frac{1728}{231} = 7.48$, or about $7\frac{1}{2}$ gallons.

455. A **Bin** for coal will hold a ton (2000 pounds)—

Of Lehigh coal for every 36 cubic feet.
Of Lackawanna coal " 38 " "
Of Franklin " " 40 " "

WRITTEN EXERCISES.

456.—1. How many bushels of corn can be put into a bin 8 feet long, 5 feet broad, and 5 feet deep?

SOLUTION.

$8 \times 5 \times 5 = 200$, *number of cubic feet.*
$200 \times .8 = 160$, *number of bushels.*

2. A rectangular cistern is 15 feet long, 10 feet broad, and 8 feet deep. How many hogsheads, of 63 gallons each, will it hold?

SOLUTION.

$15 \times 10 \times 8 = 1200$, *number of cubic feet.*
$1200 \times 7\frac{1}{2} = 9000$, *number of gallons.*
$9000 \div 63 = 142\frac{6}{7}$, *number of hogsheads.*

3. I have a bin 8 feet long, 6 feet broad, and 5 feet deep. How many tons of Franklin coal will it hold?

4. A bin will exactly contain 160 bushels of wheat. What is its capacity in cubic feet?

5. I have a cistern 6 feet long, 4 feet wide, and 5 feet deep. How many hogsheads, of 63 gallons each, is its capacity?

6. A vat will hold exactly 3840 gallons. What is its capacity in cubic feet?

457. Rules for Estimating Bins, Vats, or Cisterns.—*1. Multiply the contents in cubic feet by .8 for bushels, or by $7\frac{1}{2}$ for gallons.*

2. Divide the capacity in bushels by .8, or in gallons by $7\frac{1}{2}$, for contents in cubic feet.

PROBLEMS.

458.—1. How many bushels of grain will fill a cubical box whose dimensions are each 5 feet?

2. A vat has 96 cubic feet of interior space. How many gallons of water will it hold?

3. A tank is 6 feet long, 4 feet 6 inches broad, and 5 feet deep. How many pounds of water will it contain, the weight of a gallon of water being $8\frac{1}{3}$ pounds?

4. A chest will contain exactly 100 bushels of grain. What are its contents in cubic feet?

5. A wagon-body is 8 feet long, 3 feet 6 inches wide, and 2 feet deep. How many bushels will it contain?

6. A bin is 6 feet long and 4 feet wide. How deep must it be to contain exactly 2 tons of Lehigh coal?

7. I have a bin which exactly holds 5 tons of Lackawanna coal. It is 10 feet long and 4 feet deep. What is its width?

8. How many bushels of wheat can be put into a bin that is 8 feet long, 6 feet 3 inches wide, and 4 feet 6 inches deep?

9. I have a bin 6 feet long, 6 feet wide, and 6 feet deep, and two others each 3 feet long, 3 feet wide, and 3 feet deep. How many more bushels will the first hold than the other two?

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459. TEST QUESTIONS.—1. What is a line? A straight line? An angle? A perpendicular line? A right angle? A rectangle? A rectangular solid?

2. What are the dimensions of a rectangular surface? The area of a rectangle? What are the dimensions of a rectangular solid? The cubic contents of a rectangular solid?

3. How many cubic inches does the standard bushel contain? The standard gallon?

MISCELLANEOUS PROBLEMS.

460. The **Articles** in parentheses denote the portions of the text for which the problems may be used as supplementary exercises.

(Art. 73.)

1. Express by figures, fifty-nine thousand nine; seven thousand eighty; and fifty-one thousand one hundred three.

2. Write and read 134640; 60041; and 4602000.

3. What is the sum of two hundred fifty-two thousand six hundred six and one hundred seventy-two thousand nine hundred seventy-five?

4. How many are 96004 — 964? 83334 — 9453?

5. If you have 5675 dollars, and should spend 4987, how much would you have left?

6. How many are 7806 + 760 + 9376 + 97?

7. How many are 95631 — 777 added to 66406 + 9972?

8. A mill had 6750 barrels of flour. To A it sold 2173 barrels, and to B 978 barrels. How much then had it unsold?

9. How many are 152445 + 707050 less 93987?

10. The coach was first made in England in the year 1564, which was 1083 years after iron shoes were first made for horses. In what year were the iron shoes first made?

11. How many are 1685 + 75 + 832 + 9675?

12. How many are 59 + 641 + 9 + 8086 + 93015?

13. How many are 19678 — 3789, less 10000 — 9889?

14. A man had 50000 dollars, and paid from it for a house 10550 dollars, for land 18075, and for goods 15787. How much money had he then left?

(**Art. 117.**)

1. How much will 15 horses cost at 225 dollars each?

2. Add 3605 to the product of 716 by 97.

3. Subtract 1148 from the product of 517 by 68.

4. A farmer bought 316 sheep at 3 dollars each, and sold 250 of them for 1000 dollars, and the remainder at a loss of a dollar each. How much did he make?

5. From 1840×18 take 2045×13.

6. How much is 13465 divided by 47?

7. A planter has 64575 gallons of molasses. How many casks of 63 gallons each will be required to hold it?

8. How many are 225×36, divided by $810 \div 15$?

9. When 64 acres of land can be bought for 1600 dollars, how many acres can be bought for 3225 dollars?

10. If the multiplier is 47 and the product is 13113, what is the multiplicand?

11. If the divisor is 71 and the quotient 1002, what is the dividend?

12. The quotient is 1365, the divisor 63, and the remainder 14. What is the dividend?

13. What is the quotient of 13675 divided by 18? By 63? By 37?

14. What is the quotient of 67350 divided by 100? By 31? By 60?

15. Bought 224 barrels of flour for 1568 dollars, and sold the same for 2128 dollars. What was the gain on each barrel?

(Art. 238.)

1. Reduce $14\frac{11}{12}$ to an improper fraction.

2. What is the value of $\frac{615}{23}$? Of $\frac{514}{8}$?

3. How many ones are there in $\frac{279}{9}$? In $\frac{693}{11}$? In $\frac{111}{17}$?

4. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$ each to twenty-fourths.

5. Arrange $\frac{4}{9}$, $\frac{8}{15}$, and $\frac{3}{5}$ in the order of their values.

6. John is $11\frac{5}{8}$ years old, and William $13\frac{7}{12}$. What is the sum of their ages?

7. A pole stands $\frac{3}{16}$ of its length in the mud, $\frac{3}{10}$ of its length in the water, and the rest above water. How much of its length is above water?

8. What is the difference between $\frac{5}{6}$ and $\frac{3}{4}$?

9. What is the difference between $4\frac{1}{7}$ and $5\frac{2}{3}$?

10. How much greater is the sum of $3\frac{1}{3}$ and $2\frac{5}{6}$ than their difference?

11. What is the difference between 18 and a fifth of fifteen-fifths?

12. At $\frac{5}{8}$ of a dollar a bushel, what will 84 bushels of potatoes cost?

13. If a boy can earn $4\frac{1}{2}$ dollars in a week, how much can he earn in $\frac{2}{3}$ of a week?

14. At $2\frac{1}{2}$ dollars a day, in how many days will a man earn $28\frac{3}{4}$ dollars?

15. How much is $\frac{1}{4}$ of $\frac{42}{67}$? $\frac{1}{9}$ of $\frac{112}{4}$?

16. What number added to $\frac{3}{5}$ will give $1\frac{19}{20}$?

17. If 5 yards of cloth cost $1\frac{7}{8}$ dollars, what part of a dollar is it a yard?

18. What fraction is equivalent to $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$?

19. A man owning $\frac{3}{4}$ of a farm sells $\frac{2}{5}$ of his share for 450 dollars. At this rate what is the value of the whole farm?

20. A certain estate is worth 24000 dollars, and $\frac{3}{7}\frac{3}{6}$ of the value of the estate is $\frac{5}{7}$ of the value of the house upon it. What is the value of the house?

21. A farmer has $36\frac{3}{5}$ bushels of corn in a bin, which is just $\frac{4}{9}$ of all he raised. How much did he raise?

22. Divide 2 by the sum of $2\frac{2}{3}$ and $\frac{4}{5}$, and to the quotient add $1\frac{2}{3} - \frac{5}{6}$.

(**Art. 334.**)

1. How many dollars will 112 bushels of apples cost at $62\frac{1}{2}$ cents a bushel?

2. Bought 2 bushels of chestnuts at $3.50 per bushel, and sold them at 8 cents a pint. How much was gained by the transaction?

3. A man is 31 years old, allowing $365\frac{1}{4}$ days to a year. How many hours has he lived?

4. The distance between two places is exactly $15\frac{1}{2}$ miles. How many yards are they apart?

5. A lot of land containing exactly 47 square rods was sold at 2 cents a square foot. How much did it sell for?

6. How many acres are 4392 square rods?

7. How many hogsheads of vinegar of 63 gallons each, at 5 cents a quart, can be bought for $75.60?

8. When hay is $25 per ton, how much is it per hundred-weight?

9. How many ounces are there in 5 tons?

10. How many acres are 25090560 square inches?

11. How many pounds are 37440 troy grains?

12. How many chests of tea of 35 pounds each, at 45 cents a pound, can be bought for $63?

(**Art. 372.**)

1. How many minutes are there in 1 week 4 days 15 hours?

2. How much is the sum of 14 lb. 3 oz. 11 pwt.; 5 lb. 7 oz. 13 pwt., and 11 oz. 17 pwt. 13 gr.?

3. From 59° 42′ 15″ take 11° 39′ 47″.

4. How many ounces are there in 13 cwt. 67 lb. 14 oz.?

5. How much must be paid for 183073 square yards of land at 50 cents a square rod?

6. A certain range of wood contains exactly 10 cords. If 2 loads, each containing 1 cord 3 cord feet, be taken from it, how much will be left? What will the remainder be worth at 75 cents per cord foot?

7. What is the quotient of 34 cd. 6 cd. ft. 4 cu. ft. divided by 4? Of 118 bu. 1 pk. 5 qt. $\div 6$?

8. What is the product of 10 reams 5 quires 13 sheets multiplied by 9? 13 A. 15 p. 3 sq. yd. $\times 10$?

9. Three thieves carry off from a house 7 silver cups, each weighing 10 oz.; 1 dozen and 9 silver forks, each weighing 2 oz. 8 pwt.; and 13 silver spoons, each weighing 3 oz. Dividing the silver among the thieves, and giving to one of them a double share, how much would each have?

10. What is the value of 1 mi. 25 rd. 2 yd. 2 ft. + 5 mi. 150 rd. 3 yd. 2 ft.?

11. What is the value of 71 gal. 3 qt. 1 pt. 3 gi. $\times 8$? Of 68 d. 18 h. 56 min. $\div 4$?

12. A dray-load is found to weigh 1 ton 4 cwt. 15 lb., and it consists of 21 boxes. What is the weight of each box?

(**Art. 407.**)

1. Write one thousand, and one ten thousandth.

2. Write fifteen, and fifteen millionths.

3. Find the sum of eleven and six hundredths, added to thirteen, and one hundred six thousandths.

4. From 93 take .0093; from 1 take .000001.

5. What is the product of 1.04 multiplied by .104?

6. Express .275 as a common fraction in its simplest form.

7. Express $\frac{3}{17}$ as a decimal of 3 orders.

8. What is the value of 17 divided by 1.7?

9. What is the value of $\left(\frac{7}{.7} \times \frac{.7}{7}\right) \times 1.3$?

10. Divide 1.56 by .005, and .0003 by .05, and find the sum.

11. Multiply 7 ten-thousandths by 15 thousandths, and divide the result by .25.

12. How many thousand feet of boards at $13.50 a thousand can be bought for $101.25?

13. A man traveled 5 days; the first day he went 16.05 miles, the second 35.16 miles, the third $21\frac{7}{100}$ miles, the fourth 11.009 miles, and the fifth $31\frac{165}{10000}$ miles. How far did he travel in all?

14. I bought a cask of refined petroleum containing 48.5 gallons. How much of it can I sell and have left 13.125 gallons?

15. A person sold .15 of an estate to one person, and then $\frac{1}{8}$ of the remainder to another person. What part of the estate did he still retain?

(**Art. 433.**)

1. How much is $5\frac{1}{2}$ % of 8670 yards? 11 % of \$125? 15 per cent. of 624 bushels?

2. What % is made by selling goods at \$76.56 which cost \$63?

3. A horse which cost \$250 was sold at \$75 above cost. What was the rate % of gain?

4. Bought goods for \$1250 and sold them for \$1206.25. What was the loss per cent.?

5. Bought coal at \$7.50 per ton, and sold it so as to gain 15 %. At what price was it sold?

6. What is the interest of \$245 for 3 years, at 7 %? For 2 years 6 months, at 7 %?

7. What is the amount of \$370 for 8 months 18 days, at 9 %?

8. What is the interest of \$1000 for 1 month 6 days, at 8 %?

9. What is the amount of \$1250 for 2 years 2 months 15 days, at 6 %?

10. What is the amount of \$965 for 3 years 6 months 24 days, at 4 %?

11. What is the interest of \$50.50 from Jan. 1, 1877, to Oct. 10, 1878, at 7 %?

12. What is the amount of \$834.80 from Feb. 15, 1877, to Nov. 21, 1878, at 6 %?

13. What is the interest of \$500 from July 1, 1877, to Jan. 16, 1878, at 7 %?

14. What is the amount of \$1540.50 from April 11 to Oct. 21, 1877, at 8 %?

15. What is the amount of \$2000 from May 3, 1877, to June 13, 1878, at 5 %?

ANSWERS.

Art. 42.
3. 164.
4. 116.
5. 96.
6. 169.
9. 88.
10. 137.
11. 149.
12. 88.

Art. 43.
1. 208.
2. 168.
3. 147.
4. 209.
5. 178.
6. 220.
7. 159.
8. 179.
9. 194.
10. 158.
11. 148.
12. 178.

Art. 45.
2. 231.
3. 570.
4. 1041.
5. 963.
6. 488.

Art. 47.
1. 8660.
2. 7886.
3. 3885.
4. 5942.
6. 2722.

Art. 48.
1. 903 dollars.
2. 806 men.
3. 881 miles.
4. 921 bushels.
5. 1301 feet.
6. 7907 tons.
7. 9919 bales.
8. 1279 horses.

Art. 49.
1. 9602.
2. 14152.
3. 425435.
4. 870 yards.
5. 9514 pounds.
6. 2296 men.
7. 1834 cords.
8. 9976.
9. 4084.
10. 25308.
11. 1597124.

Art. 50.
1. 1799.
2. 6125 dollars.
3. 1045.
4. 10196.
5. 5649.
6. 573 dollars.

Art. 51.
1. 204.
2. 281.
3. 427.
4. 2068.
5. 15155.
6. 157.
7. 341.
8. 544.
9. 2465.
10. 7606.

Art. 66.
2. 1716.
3. 4899.
4. 5594.
6. 7099.
7. 8902.
8. 5200.

Art. 68.
1. 918.
2. 538.
3. 18888.
4. 81907.
5. 1001.
6. 19497.

Art. 69.
1. 42169.
2. 5700.
3. 4785.
4. 1858.
5. 3630.
6. 10375.
7. 2195.
8. 16911.

Art. 70.
1. 61.
2. 3110 dollars.
3. 811.
4. 3268 feet.
5. 1785 dollars.
6. 910; 989.
7. 18981.
8. 127987.

Art. 73.
1. 2340.
2. 3334.
3. 834.
4. 4110.
5. 11567.
6. 7457.
8. 2275.
9. 3572.
10. 4925.

Art. 86.
2. 1250.
3. 5360.
4. 2660.
5. 3100.
6. 31000.
7. 86800.

Art. 88.
2. 2996.
3. 14761.
4. 1824.
5. 9512.
6. 14652.
7. 7140.

Art. 90.

1. 7506 dollars.
2. 3612 hogsheads.
3. 7259 yards.
4. 36081 bushels.
5. 9855 days.
6. 2494 miles.
7. 731000 tons.
8. 376800 acres.

Art. 91.

2. 158632.
3. 2415207.
4. 1010025.
5. 22612390.
6. 13500 dollars.
7. 21400.
8. 210511.

Art. 92.

1. 4100 dollars.
2. 13104.
3. 117600.
4. 293760.
5. 12769.
6. 277008.
7. 146520.
8. 154350.
9. 2199582.
10. 57600000.

Art. 110.

2. 108.
3. 448.
4. 137.
5. 141.
6. 143.
7. 131.
8. 117.
9. 398.
10. 72.

Art. 112.

1. $681\frac{1}{2}$.
2. $575\frac{2}{8}$.
3. $100\frac{3}{7}$.
4. 515.
5. $332\frac{3}{14}$.
6. $260\frac{10}{15}$.
7. $21\frac{2}{21}$.
8. $558\frac{6}{12}$.

Art. 113.

1. $182\frac{3}{4}$ apples.
2. $187\frac{2}{3}$ cents.
3. $133\frac{4}{5}$ feet.
4. $59\frac{2}{7}$ rods.
5. $69\frac{8}{9}$ dollars.
6. $578\frac{5}{11}$ days.
7. $210\frac{11}{13}$ pounds.
8. $255\frac{5}{7}$ dollars.
9. 320 cents.
10. $483\frac{3}{8}$.

Art. 114.

1. $358\frac{7}{10}$.
2. 406 dollars.
3. 49.
4. 62.
5. 41.
6. 62, and 7 gallons will remain.
7. 36, and 5 acres will remain.
8. 57, and 67 feet will remain.

Art. 117.

1. 25 dollars.
2. 9443€.
3. 1012 dollars.
4. 910.
5. 4350 dollars.
6. $191\frac{8}{12}$ dollars.
7. 262 bu. of oats, 393 bu. of wheat.
8. $54\frac{20}{40}$.
9. 1775 days.
10. 4681 dollars.
11. 1800 dollars.
12. $7\frac{204}{408}$.
13. 51.

Art. 126.

2. 2, 2, 3, 7.
3. 3, 5, 5.
4. 2, 2, 2, 2, 2, 3.

Art. 128.

1. 5, 19.
2. 3, 3, 7.
3. 2, 2, 23.
4. 2, 61.
5. 2, 2, 29.
6. 2, 2, 2, 23.
7. 2, 2, 3, 3, 3.
8. 2, 2, 2, 5, 5.
9. 2, 2, 2, 91.
10. 2, 2, 37.
11. 2, 3, 5, 7.
12. 3, 5, 7, 7.

Art. 134.

2. 9.
3. 7.
4. 7.

Art. 144.

1. 144.
2. 108.
3. 168.
4. 42.
5. 126.
6. 57.
7. 120.
8. 240.
9. 560.
10. 60.

Art. 148.

3. $25\frac{2}{3}$.
4. 5, 4.

Art. 150.

1. $2\frac{4}{7}$.
2. 5.
3. 15.
4. $4\frac{3}{7}$.
5. 2.
6. 6.
7. 27.
8. 5.
10. $6\frac{3}{7}$.
11. 19.
12. $4\frac{15}{30}$.

Art. 168.

2. $\frac{76}{4}$; $\frac{250}{10}$.
3. $\frac{455}{7}$; $\frac{584}{8}$.
5. $\frac{69}{5}$; $\frac{217}{11}$.
6. $\frac{82}{3}$; $\frac{746}{13}$.

Art. 170.

1. $\frac{13}{3}$.
2. $\frac{67}{9}$.

3. $1\frac{2}{11}\frac{6}{}$.
4. $1\frac{0}{7}\frac{9}{}$.
5. $2\frac{1}{1}\frac{1}{3}$.
6. $2\frac{8}{1}\frac{9}{5}$.
7. $2\frac{1}{7}\frac{8}{}$.
8. $2\frac{0}{1}\frac{1}{0}$.
9. $3\frac{4}{1}\frac{3}{1}$.
10. $1\frac{2}{2}\frac{3}{}$.
11. $2\frac{0}{1}\frac{9}{5}$.
12. $3\frac{0}{1}\frac{6}{9}$.

Art. 172.

2. 91.
3. 73.
5. $13\frac{5}{14}$.
6. $17\frac{2}{13}$.

Art. 174.

1. 14.
2. $26\frac{3}{5}$.
3. 17.
4. $80\frac{1}{5}$.
5. $16\frac{3}{9}$.
6 $25\frac{3}{5}$.
7. 79.
8. 31.
9. $24\frac{13}{21}$.
10. 21.
11. $304\frac{1}{8}$.
12. $72\frac{5}{37}$.
13. 113; $78\frac{3}{4}$ yards.

Art. 178.

2. $\frac{42}{48}$; $\frac{12}{64}$.
3. $\frac{25}{55}$; $\frac{35}{75}$.

Art. 180.

1. $\frac{63}{72}$.
2. $\frac{10}{60}$.
3. $\frac{45}{81}$.
4. $\frac{20}{65}$.
5. $\frac{3}{57}$.
6. $\frac{36}{108}$.

Art. 184.

2. $\frac{7}{8}$; $\frac{8}{16}$.
3. $\frac{5}{11}$; $\frac{7}{15}$.

Art. 186.

1. $\frac{7}{8}$.
2. $\frac{1}{6}$.
3. $\frac{5}{9}$.
4. $\frac{4}{13}$.
5. $\frac{1}{19}$.
6. $\frac{1}{8}$.
7. $\frac{1}{9}$.
8. $\frac{1}{5}$.

Art. 189.

3. $\frac{55}{66}$; $\frac{54}{66}$.
4. $\frac{6}{18}$; $\frac{4}{18}$; $\frac{15}{18}$.

Art. 192.

1. $\frac{9}{12}$; $\frac{10}{12}$.
2. $\frac{18}{30}$; $\frac{3}{30}$; $\frac{8}{30}$.
3. $\frac{48}{88}$; $\frac{20}{88}$; $\frac{14}{88}$.
4. $\frac{24}{36}$; $\frac{16}{36}$; $\frac{21}{36}$.
5. $\frac{12}{24}$; $\frac{20}{24}$; $\frac{9}{24}$.
6. $\frac{16}{40}$; $\frac{35}{40}$; $\frac{36}{40}$.
7. $\frac{20}{70}$; $\frac{21}{70}$; $\frac{55}{70}$.
8. $\frac{7}{42}$; $\frac{27}{42}$; $\frac{16}{42}$.
9. $\frac{32}{40}$; $\frac{35}{40}$; $\frac{30}{40}$.
10. $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$.
11. $\frac{24}{180}$; $\frac{110}{180}$; $\frac{153}{180}$.
12. $\frac{495}{720}$; $\frac{180}{720}$; $\frac{336}{720}$.

Art. 196.

2. $2\frac{7}{20}$.
3. $2\frac{7}{32}$.
5. $2\frac{109}{504}$.
6. $2\frac{13}{30}$.
8. $33\frac{1}{8}$.

Art. 198.

1. $2\frac{1}{80}$.
2. $\frac{95}{112}$.
3. $1\frac{11}{25}$.
4. $1\frac{1}{60}$.
5. $5\frac{67}{72}$.
6. $10\frac{7}{22}$.
7. $40\frac{7}{10}$.
8. $2\frac{13}{24}$.
9. $3\frac{2}{5}$.
10. $7\frac{1}{3}$.
11. $63\frac{5}{8}$.
12. $20\frac{17}{30}$.

Art. 201.

2. $\frac{7}{28}$.
3. $\frac{59}{104}$.
5. $7\frac{9}{20}$.
6. $19\frac{5}{14}$.

Art. 203.

1. $\frac{5}{24}$.
2. $\frac{3}{18}$.
3. $\frac{9}{62}$.
4. $\frac{18}{42}$.
5. $\frac{5}{132}$.
6. $\frac{9}{34}$.
7. $3\frac{5}{36}$.
8. $8\frac{1}{6}$.
9. $66\frac{1}{16}$.
10. $42\frac{5}{16}$.
11. $2\frac{3}{10}$.

Art. 207.

3. $2\frac{1}{9}$; $7\frac{24}{101}$; $5\frac{3}{20}$.
4. 414.

Art. 209.

1. $6\frac{3}{10}$.
2. $16\frac{5}{13}$.
3. $19\frac{4}{9}$.
4. $1\frac{1}{2}$.
5. $4\frac{2}{3}$.
6. $2\frac{5}{7}$.
7. $2\frac{1}{2}$.
8. $3\frac{33}{91}$.
9. $60\frac{4}{5}$.
10. 104.
11. $129\frac{7}{11}$.
12. 1215.

Art. 214.

3. $30\frac{15}{17}$.
4. $49\frac{7}{9}$.
5. $25\frac{5}{17}$.
6. 280.
7. $102\frac{3}{16}$.
8. 387.

9. 78.

Art. 216.

3. $\frac{12}{85}$.
4. $36\frac{1}{3}$; $181\frac{9}{11}$.

Art. 218.

1. $\frac{5}{27}$.
2. $\frac{21}{124}$.
3. $\frac{5}{19}$.
4. $\frac{1}{5}$.
5. $\frac{5}{5}\frac{2}{2}\frac{5}{8}\frac{5}{7}$.
6. $2\frac{11}{12}\frac{5}{8}$.
7. $10\frac{11}{12}$.
8. $2\frac{8}{27}$.
9. $1\frac{3}{8}\frac{8}{2}\frac{7}{5}$.
10. $\frac{33}{63}$.
11. 20.
12. $89\frac{3}{5}$.
13. $\frac{10}{12}\frac{5}{8}$.
14. $\frac{1}{3}$.
15. $\frac{1}{5}$.
16. $2\frac{4}{5}$ dollars.

Art. 222.

3. $\frac{3}{31}$; $\frac{1}{124}$; $3\frac{9}{28}$.

Art. 224.

1. $\frac{7}{48}$.
2. $\frac{1}{12}$.
3. $\frac{1}{24}$.
4. $\frac{1}{63}$.
5. $\frac{2}{47}$.
6. $\frac{2}{19}$.
7. $2\frac{5}{14}$.
8. $2\frac{11}{32}$.
9. $1\frac{13}{50}$.
10. $8\frac{6}{25}$ dollars.

Art. 226.

2. $73\frac{1}{7}$.
3. $33\frac{1}{3}$; $134\frac{2}{5}$; $220\frac{1}{2}$.

Art. 228.

1. $26\frac{1}{4}$.
2. $91\frac{1}{2}$.
3. $52\frac{1}{2}$.
4. 64.
5. 867.
6. $49\frac{1}{7}$.
7. $238\frac{1}{3}$.
8. $337\frac{1}{2}$.
9. 84.
10. $40\frac{4}{5}$.
11. 81.
12. 200.
14. 6; $17\frac{9}{11}$; $6\frac{3}{10}$.
15. 10.

Art. 231.

2. $1\frac{9}{24}$.
3. $8\frac{1}{6}$.

Art. 233.

1. 4.
2. $1\frac{1}{10}$.
3. $1\frac{1}{14}$.
4. $1\frac{7}{10}$.
5. $2\frac{4}{5}$.
10. 5.
11. $\frac{12}{23}$.
12. $\frac{1}{4}$.
14. 6.
15. $4\frac{2}{3}$.
16. $4\frac{2}{3}$; $4\frac{1}{4}$.
17. $3\frac{1}{2}$.

Art. 236.

1. $\frac{8}{8}\frac{4}{8}\frac{3}{8}$.
2. $\frac{14}{36}$; $\frac{15}{36}$.
3. $\frac{2}{3}$; $\frac{7}{9}$.
4. $14\frac{3}{8}$ dollars.
5. $\frac{24}{84}$.
6. $26\frac{1}{4}$; $31\frac{1}{2}$.
7. $2\frac{9}{11}$; $2\frac{2}{11}$.
8. $5\frac{4}{19}$; $5\frac{15}{19}$.
9. $2\frac{1}{2}$ tons.
10. $16\frac{1}{8}$ dollars.

Art. 238.

1. 26 yards.
2. $\frac{7}{8}\frac{3}{8}$; $\frac{11}{2}\frac{0}{0}\frac{4}{4}\frac{3}{8}$; 4.
3. 11.
4. 126 dollars.
5. $4\frac{4}{5}$.
6. $15\frac{3}{5}$ dollars.
7. $4\frac{8}{9}$.
8. $\frac{4}{17}$.
9. $14\frac{2}{3}$.
10. 640.

Art. 253.

3. $16.25
4. $82.28
6. $262.37
7. $7015.50

Art. 255.

2. $77.06
3. $77.915
4. $26.98
5. $103.81
6. $3688.18
7. $83.50

Art. 257.

2. $490.72
3. $447.15
4. $78.125
5. $228.72
6. $260.169
7. $9356.
8. $13065.
9. $180576.
10. $34055.
11. $10303.
12. $391.80
13. $17503.50
14. $1510.44
15. $76800.

Art. 259.

3. $40.65
4. $6.10
5. $25.06
6. $6.12
7. $93.56
8. $1005.
9. $3.64
10. $634.055

11. $56.75
12. $1.31
13. $19.04
14. $1111.44
15. $16.75
17. 20.
18. $5.25
19. 85.
20. $2641.
21. $12.

Art. 262.

2. $90.
3. $63.
4. $8.

Art. 264.

1. $947.50
2. $66.75
3. $8510.
4. $32.

Art. 270.

Bill receipted, $21.37½.

Art. 277.

5. 15; 99.

Art. 287.

9. 155520.
10. 36.
11. 65.

Art. 300.

5. $30.24
6. $9.60

Art. 303.

6. 6.

Art. 312.

7. 480.

Art. 323.

5. 2400.
6. 72.
7. 12600.

Art. 327.

8. 366.
9. 5785560.

Art. 331.

4. 2544.
5. $51.60
6. 24.
7. $72.
8. $450.

Art. 334.

1. 527040.
2. $165.
3. 12.
6. $403.20
7. 23040.
8. 40.
9. 40.

Art. 341.

2. 175 in.
3. 7515 sq. in.

Art. 343.

1. 11056″.
2. 18755 sq. yd.
3. 7569 ft.
4. 4700260 cu. in.
5. 253 qt.
6. 4601 gr.
7. 142544 oz.
8. 335 qt.
9. $300.
10. $504.
11. $44.
12. 441300 h.

Art. 345.

2. 4 yd. 2 ft. 7 in.
3. 5 sq. yd. 7 sq. ft. 27 sq. in.
4. 10 h. 17 m. 18 sec.

Art. 347.

1. 3° 4′ 16″.
2. 3 A. 140 P.
3. 1 mi. 138 rd. 4 yd.
4. 100 cu. yd. 20 cu. ft. 100 cu. in.
5. 8 bu. 2 pk. 3 qt.
6. 9 oz. 11 pwt. 17 gr.
7. 4 T. 9 cwt. 5 lb. 14 oz.
8. 1 hhd. 20 gal. 3 qt.
9. 1 A. 40 sq. rd.
10. 1 mi. 100 rd.
11. 1ʒ 6ʒ 2ϑ.
12. 50 y.

Art. 351.

2. 14 mi. 17 rd.
3. 23 gal. 2 qt.

Art. 353.

1. 19 cwt. 1 lb. 5 oz.
2. 87 A. 76 P.
3. 20 cu. yd. 21 cu. ft. 28 cu. in.
4. 1 lb. 9 oz. 19 pwt. 6 gr.
5. 191 bu. 3 pk.
6. 54 rd. 2 yd. 2 ft.
7. 15 d. 23 h. 37 m.
8. 52° 55′ 48″.
9. 1 bu. 1 pk. 3 qt. 1 pt.
10. 33 A. 100 sq. rd.
11. 47 T. 8 cwt. 56 lb.

Art. 356.

2. 3 lb. 10 oz. 2 pwt.
3. 1 gal. 2 qt. 1 pt.
4. 8 yd. 2 qr. 3 na.

Art. 358.

1. 8 m. 300 rd. 1 yd.
2. 2 A. 155 P. 5 sq. yd.
3. 1 wk. 4 d. 23 h.
4. 22 T. 19 cwt. 94 lb. 11oz.
5. 3ʒ 5ʒ 1ϑ.
6. 1° 51′ 15″.
7. 19 m. 34 rd. 2 yd. 1 ft. 6 in.
8. 1 y. 10 mo.
9. 28 gal. 1 pt.
10. 16° 34′ 17″.
12. 52 y. 1 mo. 18 d.

Art. 362.

2. 32 rd. 0 yd. 2 ft.
3. 81 lb. 5 pwt. 16 gr.

Art. 364.

1. 8 hhd. 12 gal. 1 qt.
2. 405 cu. ft. 328 cu. in.
3. 57 A. 92 P. 4 sq. yd.
4. 43 bu. 1 pk. 2 qt.
5. 64 wk. 6 d. 4 h.
6. 104° 1′ 52″.
7. 15 cd. 6 cd. ft.
8. 9 cwt. 20 lb. 8 oz.
9. 41 y. 3 mo.
10. 301 mi. 44 rd.
11. 281 A. 85 P.
12. 11 hhd. 61 gal. 2 qt.
13. 2 lb. 3 oz. 11 pwt. 6 gr.
14. 336 A. 24 P.

Art. 367.

2. 6 rd. 2 yd. 0 ft. $9\frac{3}{5}$ in.
3. 10 lb. 1 oz. 10 pwt. 17 gr.

Art. 369.

1. 1 hhd. 6 gal. 1 qt. 1 pt. $2\frac{8}{7}$ gi.
2. 50 cu. ft. 1121 cu. in.
3. 3 A. 95 P. 22 sq. yd. $8\frac{7}{16}$ sq. ft.
4. 4 bu. 0 pk. 4 qt. 1 pt. $1\frac{1}{8}$ gi.
5. 3 cd. 2 cd. ft. 5 cu. ft. 576 cu. in.
6. 8 y. 0 mo. 18 d.
7. 1 cwt. 3 lb. $14\frac{6}{7}$ oz.
8. 28 mi. 4 rd.
9. 40 A. 35 P.
10. 1 oz. 5 pwt. $12\frac{5}{9}$ gr.
11. 35 bags.
12. 1 A. 61 P. 10 sq. yd. 108 sq. in.
13. 7 h. 45 min. 50 sec.

Art. 372.

1. 1793 steps.
2. 1 mi. 6 rd.
3. 28 lots.
4. 63000″.
5. $2\frac{1}{4}$ cords.
6. 16 spoons; and 16 pwt. will remain.
7. 15 days.
8. $43.20
9. $203.40
10. $20\frac{3}{5}$ loads.
11. $9\frac{93}{101}$ hours.
12. 10.

Art. 379.

3. Three thousand one hundred sixty-five ten thousandths.
4. Five hundred fifteen, and six thousandths.
6. .1331
7. 11011.011

Art. 381.

1. Three hundred forty-seven thousandths.
2. One thousand four hundred five ten-thousandths.
3. Seventy-two ten-thousandths.
4. One hundred thirty-eight thousandths.

5. Ten thousand sixty-five hundred thousandths.
6. Two millionths.
7. Fourteen, and five thousandths.
8. Nine, and ninety-three thousandths.
9. One, and six thousand four hundred forty-four ten-thousandths.
10. .5
11. .31
12. .04
13. .017
14. .434
15. .1111
16. 14.05
17. 9.003
18. 1.06444
19. .98
20. .125
21. .1018
22. 6.75
23. 2462.7
24. 400.044
25. 24.000024

Art. 384.

2. 80.928
4. 22.44
5. 4.257

Art. 386.

1. 16.275
2. 41.151
3. 1.182
4. 34.039
5. 8.505
6. .137 of a ton.
7. 97.02
8. 121.875
9. 114.028
10. $206.

Art. 390.

3. 2.865
4. .0450

Art. 392.

1. 17.5
2. 1.75
3. 17.5
4. 1.75
5. 3.01
6. .0301
7. 30.1
8. 30.1
9. 4.872
10. .4872
11. 4.872
12. 48.72
13. 7.5
14. 215042.
15. 3.65
16. $34.85

Art. 395.

3. 3.15
   1.34

Art. 397.

1. 1.23
2. 43.
3. 3.3
4. 3.3833+
5. .05
6. 100.
7. .01
8. 10.
9. .42
10. 75.
11. 10.
12. .123
13. .756
14. .03145
15. .0000905
17. .0609+
18. 15.5 yd.

Art. 399.

2. $\frac{1}{20}$.
3. $\frac{2}{125}$.
4. $\frac{1}{8}$.
5. $\frac{5}{8}$.

Art. 401.

1. $\frac{17}{20}$.
2. $\frac{1}{8}$.
3. $\frac{1}{40}$.
4. $\frac{2}{125}$.
5. $\frac{5}{8}$.
6. $\frac{1}{80}$.
7. $\frac{12}{125}$.
8. $\frac{1}{125}$.
9. $1\frac{11}{40}$.

Art. 403.

2. .75
3. .08

Art. 405.

1. .85
2. .125
3. .025
4. .016
5. .625
6. .0625
7. .096
8. .008
9. 1.275
11. .3846+

Art. 407.

1. $\frac{3}{80}$.
2. .0017
3. $\frac{1}{8}$.
4. .368+
5. $55.25
6. 31.856
7. 20.4
8. $507.78
9. $60.477
10. 5305.
11. 100.
12. 2146.85 pounds.

Art. 414.

1. .03
2. .04
3. .07
4. .09
5. .04$\frac{3}{4}$
6. .09$\frac{1}{2}$
7. .17$\frac{1}{4}$

8. .07$\frac{1}{8}$
9. .45
10. .67
11. 1.25
12. 1.35

Art. 416.

2. 43 pounds.
3. 90 men.

Art. 418.

1. $2.46
2. $34.09
3. 11.6 yd.
4. 132.5 rd.
5. 160.3
6. 27.625
7. 106.78
8. 105.6
9. $75.
10. 36 men.
11. 84 cwt.
12. 435 lbs.
13. $135.
14. $187.50
15. 2176 bu.
16. 11448.

Art. 420.

2. 5%.
3. $3\frac{15}{19}$%.

Art. 422.

1. $16\frac{2}{3}$%.
2. $6\frac{2}{5}$%.
3. $6\frac{1}{26}$%.
4. 25%.
5. $1\frac{68}{91}\frac{7}{6}$%.
6. 7%.
7. 19%.
8. 60%.
9. $9\frac{5}{6}\frac{8}{3}$%.
10. 8%.
11. 15%.
12. 60%.
13. 45%.
14. 5%.
15. $27\frac{7}{9}\frac{0}{9}\frac{9}{9}$%.
16. 40%.
17. 20%.

Art. 428.

2. $192.50
3. $265.
4. $168.
5. $191.67
7. $618.90
8. $727.56
9. $27.778
10. $12.60

Art. 430.

1. $472.50
2. $3.105
3. 41.777
4. $1236.667
5. $693.333
6. $5279.167
7. $18.36
8. $166.625
9. $.588
10. $813.61
11. $965.08
12. $300.90
13. $55.026
14. $1099.75
15. $1368.

Art. 433.

1. $37.50
2. $6.681
3. $1.89
4. $5000.
5. $1095.85
6. $949.333
7. $12.60
8. 10%.
9. $495.
10. $300.
11. $1854.16
12. $5205.493

ANSWERS TO APPENDIX.

Art. 444.

3. 76 ft.
4. 45.

Art. 446.

1. 1400 sq. rd.
2. 1849 sq. ft.
3. 43 rd.
4. 15 rd.
5. 40 sq. yd.

Art. 450.

4. 7.

Art. 452.

1. 216.
2. 1.
3. 1500.
4. 5280.
5. 2.
6. $7.20

Art. 456.

3. 6.
4. 200.
5. $14\frac{2}{7}$.
6. 512.

Art. 458.

1. 100.
2. 720.
3. $8437\frac{1}{2}$.
4. 125.
5. 44.8
6. 3 ft.
7. 4 ft. 9 in.
8. 180.
9. 129.6

ANSWERS TO MISCELLANEOUS PROBLEMS.

Art. 73.

1. 59009.
   7080.
   51103.
3. 425581.
4. 95040.
   73881.
5. 688 dollars.
6. 18039.
7. 171232.
8. 3599 barrels.
9. 765508.
10. 481.
11. 12267.
12. 101810.
13. 15778.
14. 5588.

Art. 117.

1. 3375 dollars.
2. 73057.
3. 34008.
4. 184 dollars.
5. 6535.
6. $286\frac{23}{47}$.
7. 1025.
8. 150.
9. 129.
10. 279.
11. 71142.
12. 86009.
13. $759\frac{13}{18}$. $217\frac{4}{63}$. $369\frac{22}{37}$.
14. $673\frac{50}{100}$. $2172\frac{18}{21}$. $1122\frac{30}{60}$.
15. $2\frac{1}{2}$ dollars.

Art. 238.

1. $\frac{179}{12}$.
2. $26\frac{17}{64}$; $64\frac{1}{4}$.
3. 31; 63; $6\frac{9}{17}$.
4. $\frac{16}{21}$; $\frac{20}{34}$; $\frac{14}{24}$.
5. $\frac{3}{5}$; $\frac{8}{15}$; $\frac{4}{6}$.
6. $25\frac{5}{24}$ years.
7. $\frac{41}{80}$.
8. $\frac{1}{1\frac{1}{2}}$.
9. $\frac{4}{2\frac{1}{4}}$.
10. $\frac{3\frac{1}{4}}{6}$.
11. $17\frac{2}{5}$.
12. $52.50
13. $3.00
14. $11\frac{1}{2}$.
15. $\frac{2\frac{1}{4}}{1\frac{3}{4}}$; $3\frac{1}{9}$.
16. $1\frac{7}{20}$.
17. $\frac{3}{8}$.
18. $\frac{1}{2}$.
19. $1500.
20. $6300.
21. $81\frac{27}{32}$ bushels.
22. $1\frac{16}{39}$.

Art. 334.

1. $70.
2. $3.24
3. 271746.
4. 27280.
5. $255.91\frac{1}{2}$
6. $27\frac{9}{20}$.
7. 6.
8. $1.25
9. 160000.
10. 4.
11. $6\frac{1}{2}$.
12. 4.

Art. 372.

1. 16740.
2. 20 lb. 11 oz. 1 pwt. 13 gr.
3. 48° 2′ 28″.
4. 21886.
5. $3026.
6. $43.50
7. 8 rd. 5 cd. ft. 9 cu. ft.; 19 bu. 2 pk. 7 qt. 1 pt.
8. 92 r. 9 q. 21 s. 130 A. 150 p. 30 yd.
9. One, 79 oz. 14 pwt.; each of the others, 39 oz. 17 pwt.
10. 6 mi. 176 rd. 0 yd. $2\frac{1}{2}$ ft.
11. 577 gal. 3 qt. 17 d. 4 h. 44 m.
12. 1 cwt. 15 lb.

Art. 407.

1. 1000.0001
2. 15.000015
3. 24.166
4. 92.9907 .999999
5. .10816
6. $\frac{11}{40}$.
7. .176
8. 10.
9. 1.3
10. 312.006
11. .000042
12. $7\frac{1}{2}$.
13. 114.3055
14. 35.375 gal.
15. .74375

Art. 433.

1. 476.85 yd.; $13.75; 9.36 bu.
2. .27 +
3. 30.
4. 35.
5. $8.62\frac{1}{2}$
6. $51.45; $42.87\frac{1}{2}$
7. $393.86\frac{1}{2}$
8. $8.
9. $1415.62\frac{1}{2}$
10. $1102.67
11. $6.27
12. $923.29
13. $18.96
14. $1605.54
15. $2111.11

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